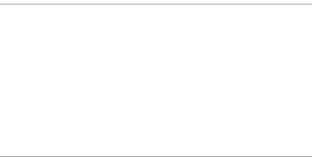


#1

SOME PROPERTIES OF LUMPED-FILTER CIRCUITS FOR TRAVELING-WAVE TUBES

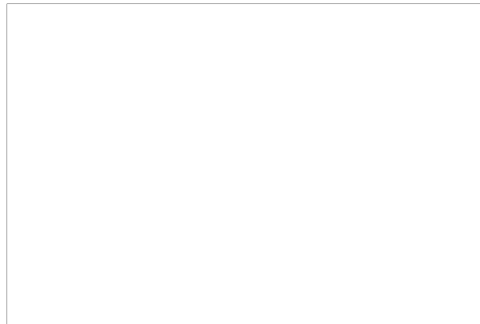
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Technical Report No. 4
July 20, 1956

NAVY Review Completed

Prepared under Office of Naval Research Contract
Nonr 225(24), NR 373 360
Jointly supported by the U.S. Army Signal Corps,
the U.S. Air Force, and the U.S. Navy
(Office of Naval Research)



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TABLE OF FIGURES OF THE
4-ELEMENT BAND PASS CIRCUIT

x_1	θ	y_{pn}	y_g	$\frac{Z_{OT}}{L_2/C_1}$	$\frac{Z_{OT}}{L_2/C_1}$	$\frac{K_{TO}}{L_2/C_1}$	$\frac{K_{TL}}{L_2/C_1}$	$\frac{K_{LO}}{L_2/C_1}$	$\frac{K_{TL}}{L_2/C_1}$
10	7.2 -69-	7.3 -70-	7.4 -71-	7.5 -72-	7.6 -73-	7.7 -74-	7.8 -75-	7.9 -76-	7.10 -77-
5	7.11 -78-	7.12 -79-	7.13 -80-	7.14 -81-	7.15 -82-	7.16 -83-	7.17 -84-	7.18 -85-	7.19 -86-
2	7.20 -87-	7.21 -88-	7.22 -89-	7.23 -90-	7.24 -91-	7.25 -92-	7.26 -93-	7.27 -94-	7.28 -95-
1.70	7.29 -96-	7.30 -97-	7.31 -98-	7.32 -99-	7.33 -100-	7.34 -101-	7.35 -102-	7.36 -103-	7.37 -104-
1.6	7.38 -105-	7.39 -106-	7.40 -107-	7.41 -108-	7.42 -109-	7.43 -110-	7.44 -111-	7.45 -112-	7.46 -113-
1.5	7.47 -114-	7.48 -115-	7.49 -116-	7.50 -117-	7.51 -118-	7.52 -119-	7.53 -120-	7.54 -121-	7.55 -122-
0.5	7.56 -123-	7.57 -124-	7.58 -125-	7.59 -126-	7.60 -127-	7.61 -128-	7.62 -129-	7.63 -130-	7.64 -131-
0.3	7.65 -132-	7.66 -133-	7.67 -134-	7.68 -135-	7.69 -136-	7.70 -137-	7.71 -138-	7.72 -139-	7.73 -140-
0.16	7.74 -141-	7.75 -142-	7.76 -143-	7.77 -144-	7.78 -145-	7.79 -146-	7.80 -147-	7.81 -148-	7.82 -149-
0.100	7.83 -150-	7.84 -151-	7.85 -152-	7.86 -153-	7.87 -154-	7.88 -155-	7.89 -156-	7.90 -157-	7.91 -158-

LIST OF SYMBOLS

b	Pierce's velocity parameter $b = \frac{u_0 - v_{pn}}{v_{pn}}$
c	velocity of light
C_n	Pierce's circuit-beam coupling parameter $C_n^2 = I_0/4V_0 K_n$
d	length of the gap
f	frequency
f_c	cut-off frequency
g	subscript for group velocity
m_r	$= 1/\sqrt{1 + 4C_1/C_2}$
m_s	$= 1/\sqrt{1 + 4L_2/L_1}$
n	index for the nth space-harmonic component
r	capacitance ratio $r = 4C_1/C_2$
s	inductance ratio $s = 4L_2/L_1$
u_0	d-c velocity of the electron beam
v_{pn}	phase velocity of the nth space harmonic
v_g	group velocity
x	normalized frequency $x = f/f_c$
x_1	one of the normalized cut-off frequencies
y_{pn}	normalized phase velocity $y_{pn} = 2x/\beta_n D$
y_g	normalized group velocity $y_g = 2dx/d\theta$
C_1	capacitance in the X_1 arm
C_2	capacitance in the X_2 arm
D	length of one space-harmonic period
E_n	magnitude of the electric field of the nth space-harmonic component
K_{Tn}	interaction impedance of the nth space-harmonic component of the transverse interaction structure
K_{Ln}	interaction impedance of the nth space-harmonic component of the longitudinal interaction structure
L_1	inductance in the X_1 arm

LIST OF SYMBOLS (Cont'd)

L_2	inductance in the X_2 arm
M_n	amplitude of the nth space-harmonic field component
P	total power carried by the circuit
V	peak voltage between line and ground
V_0	d-c beam voltage
V_n	synchronism voltage of the nth space harmonic component
	$V_0 = V_n (1 + bC_n)^2$
	$V_n = 2800 \left[y_{pn} f_c (\text{Mc/sec}) D (\text{cm}) \right]^2 \text{ volts}$
	$= 18100 \left[y_{pn} f_c (\text{kMc/sec}) D (\text{inch}) \right]^2 \text{ volts}$
X_1	reactance in arm 1 or the series arm
X_2	reactance in arm 2 or the shunt arm
Z_{OT}	characteristic circuit impedance of a T filter section
Z_{OP}	characteristic circuit impedance of a Pi filter section
β_n	wave number of the nth space harmonic component $\beta_n D = \theta + 2n\pi$
θ	circuit phase constant
ω	angular frequency $\omega = 2\pi f$

1. INTRODUCTION

Since the development of the external-circuit traveling-wave tube, many different types of slow-wave circuits can be used with a traveling-wave tube structure with a single basic interaction geometry containing the electron beam and the interaction electrodes. To use such a versatile device it becomes necessary to investigate theoretically the properties of the various possible external circuits. Some sample calculations were obtained for the various types of circuits, either lumped or distributed-lumped, which can be used as slow-wave structures.¹

In this report, results of more detailed calculations for several lumped-filter circuits are presented in normalized graphic form. The circuits include the low-pass, the high-pass,* the three-element inductive band-pass,* the three-element capacitive band-pass and the four-element band-pass circuits.

The characteristics presented include the phase-frequency characteristics, the phase velocity characteristics, the group velocity characteristics, the circuit impedance characteristics and the interaction impedance characteristics.

The interaction geometry considered here is uniformly periodic, i.e. all cylinders have equal lengths and all gaps have equal lengths. The quantities investigated are all normalized and plotted as a function of the normalized frequency.

* These circuits can never be obtained physically in their exact forms due to distributed coil capacitance and lead inductance.

II. GENERAL EQUATIONS

In this section a list of formulae for Tee and Pi filter circuits shown in Fig. 2.1 is given in terms of the reactances X_1 and X_2 and of the normalized frequency x . The circuit properties are

phase constant

$$\theta = \pm 2 \sin^{-1} \sqrt{\frac{X_1}{4X_2}} \quad (2.1)$$

phase velocity

$$v_{pn} = \frac{2x}{\theta + 2n\pi} \quad (2.2)$$

group velocity

$$v_g = 2 \frac{dx}{d\theta} \quad (2.3)$$

circuit impedances

$$Z_{OT} = \frac{\sqrt{X_1 X_2}}{\sqrt{1 + \frac{X_1}{4X_2}}} \quad (2.4)$$

$$Z_{OI} = \sqrt{X_1 X_2} \sqrt{1 + \frac{X_1}{4X_2}} \quad (2.5)$$

The interaction impedance is defined by

$$K_n = \frac{M_n^2 V^2}{\beta_n^2 D^2 2P} \quad (2.6)$$

where the amplitude factor of the nth space harmonics is¹

$$M_n^2 = 4 \sin^2 \frac{\beta_n D}{4} \quad \text{transversely loaded} \quad (2.7)$$

$$M_n^2 = 4 \sin^2 \frac{\beta_n D}{2} \quad \text{longitudinally loaded} \quad (2.8)$$

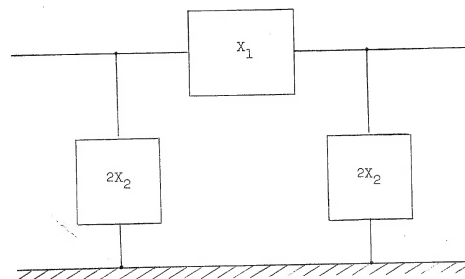
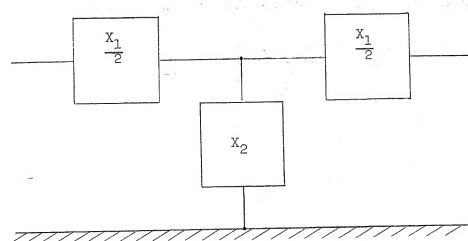


FIG. 2.1.--Tee and Pi filter sections.

Thus the longitudinal interaction impedance becomes

$$K_{Ln} = Z_{O\pi} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (2.9)$$

and the transverse interaction impedance becomes

$$K_{Tn} = Z_{O\pi} \frac{1}{4} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (2.10)$$

III. CHARACTERISTICS OF THE LOW-PASS CIRCUITS

The low-pass circuits are shown in Fig. 3.1. The first two have transverse interaction gaps and the third one has longitudinal interaction gaps.

The equations for the characteristics are

$$\theta = 2\sin^{-1} \frac{mx}{\sqrt{1-(1-m^2)x^2}} \quad (3.1)$$

$$y_{pn} = \frac{2x}{\theta + 2\pi n} \quad (3.2)$$

$$y_g = \frac{\sqrt{1-x^2}}{m} [1-(1-m^2)x^2] \quad (3.3)$$

$$Z_{O\pi} = \sqrt{\frac{L_1}{C_2}} \frac{1-x^2(1-m^2)}{\sqrt{1-x^2}} \quad (3.4)$$

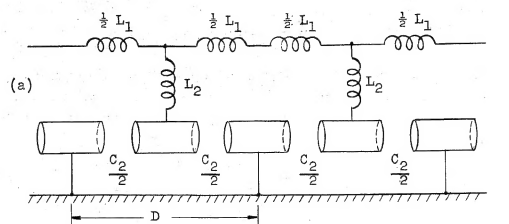
$$Z_{OT} = \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \quad (3.5)$$

$$Z_{OT} = \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-x^2}}{1-x^2(1-m^2)} \quad (3.6)$$

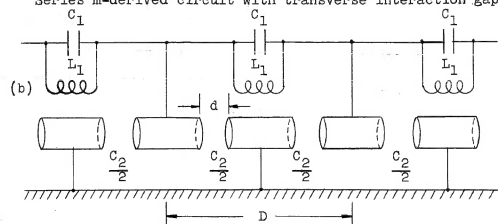
$$Z_{O\pi} = \sqrt{\frac{L_1}{C_2}} \frac{1}{\sqrt{1-x^2}} \quad (3.7)$$

$$K_{Tn} = \sqrt{\frac{L_1}{C_2}} \frac{1}{4} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{1}{\sqrt{1-x^2} [1-x^2(1-m^2)]} \quad (3.8)$$

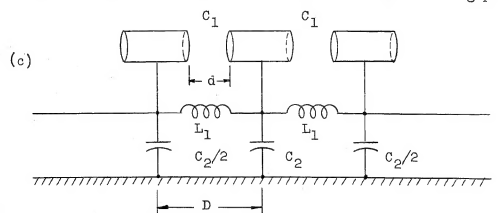
Fig. 2(a)



Series m-derived circuit with transverse interaction gaps



Shunt m-derived circuit with transverse interaction gaps



Series m-derived circuit with longitudinal interaction gaps

FIG. 3.1.--M-derived low-pass circuits.

$$K_{Tn} = \sqrt{\frac{L_1}{C_2}} \frac{1}{4} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{1}{\sqrt{1-x^2}} \quad \text{Fig. 2(b)} \quad (3.9)$$

$$K_{Ln} = \sqrt{\frac{L_1}{C_2}} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \frac{1}{\sqrt{1-x^2}} \quad \text{Fig. 2(c)} \quad (3.10)$$

These equations are plotted as curves in Figs. 3.2 to 3.12.

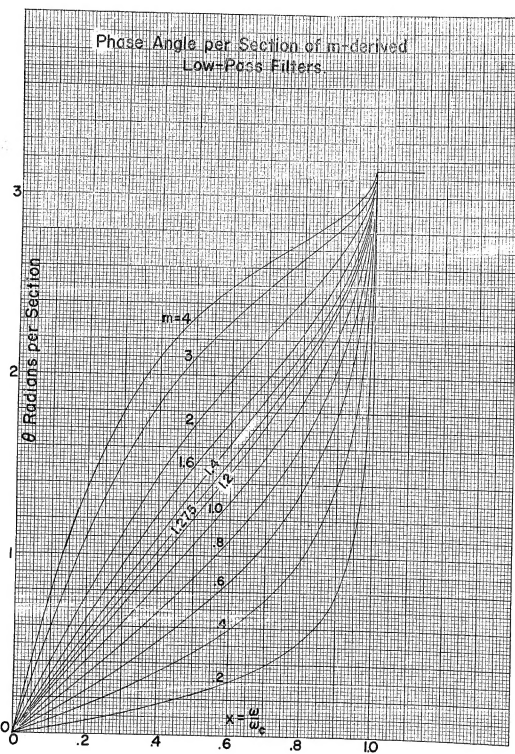


FIG. 3.2.--Phase angle per section of low-pass circuits.

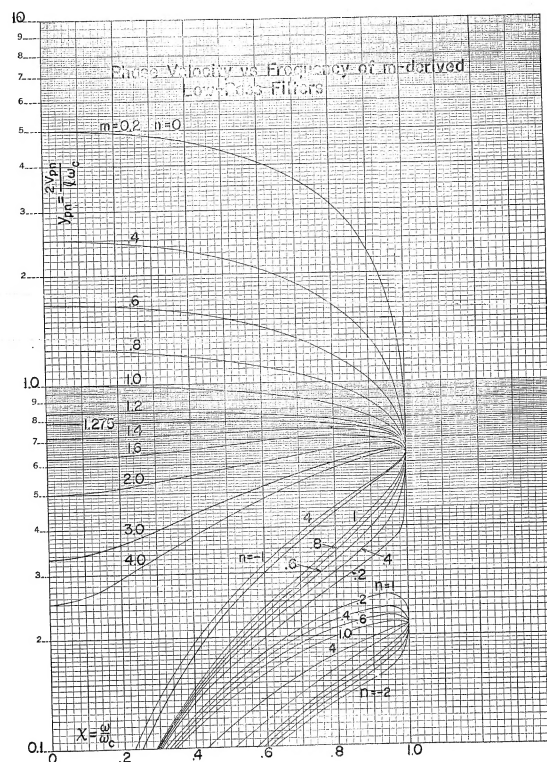


FIG. 3.3.--Phase velocity of low-pass circuits.

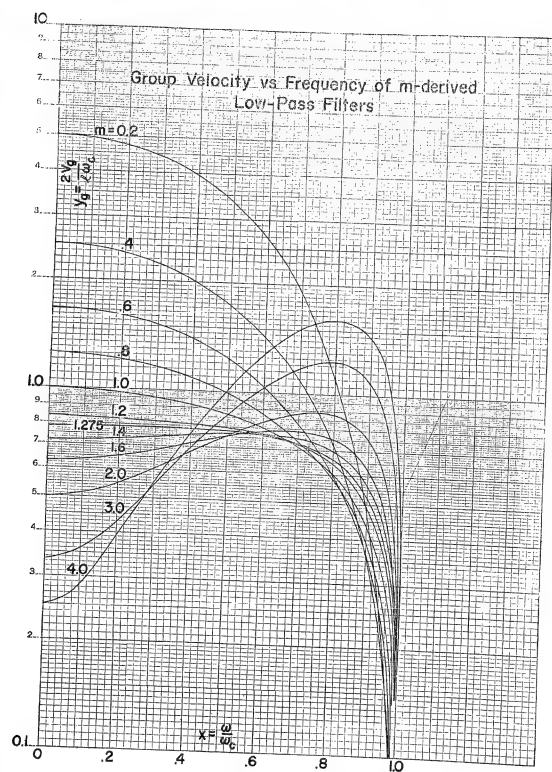


FIG. 3.4.--Group velocity of low-pass circuits.

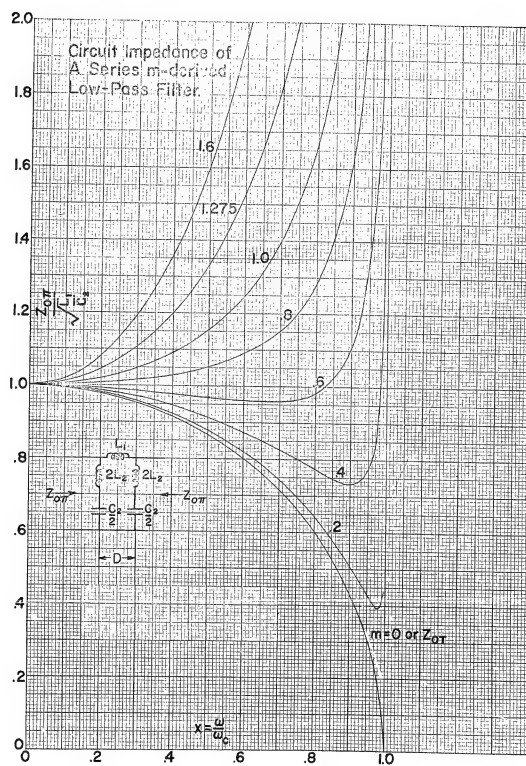
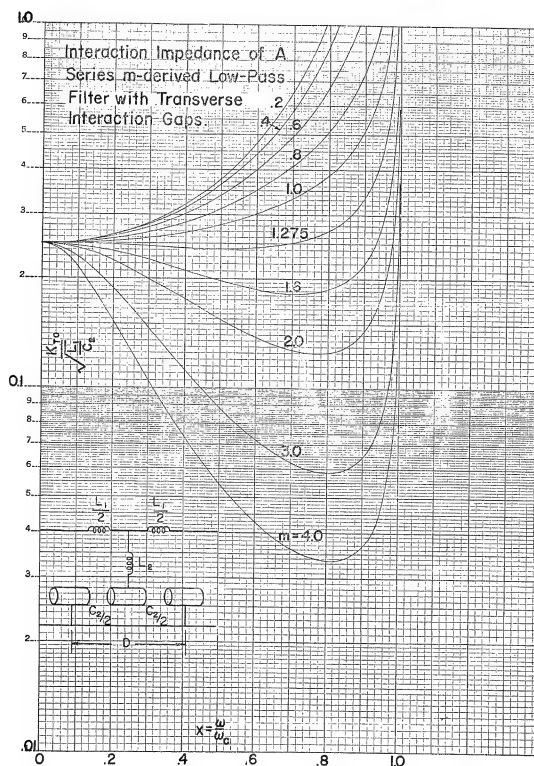
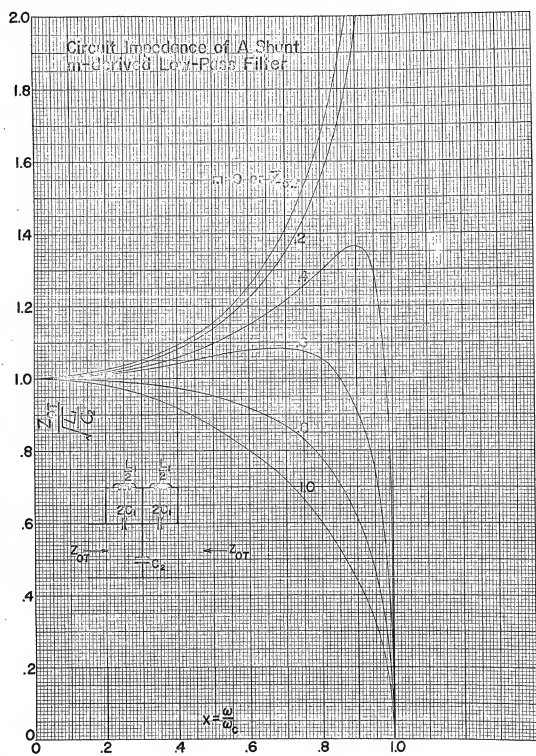


FIG. 3.5.--Circuit impedance of low-pass circuits of Pi section.



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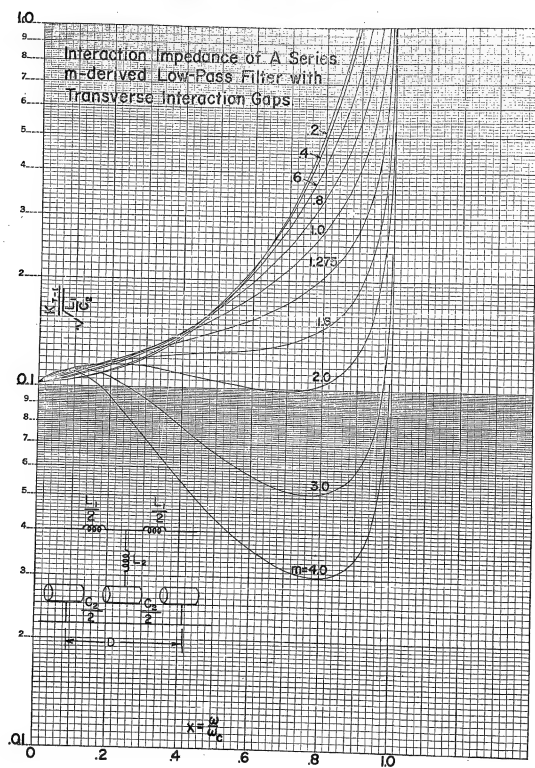


FIG. 3.5.--Transverse interaction impedance K_{T-1} of a series m-derived low-pass circuit.

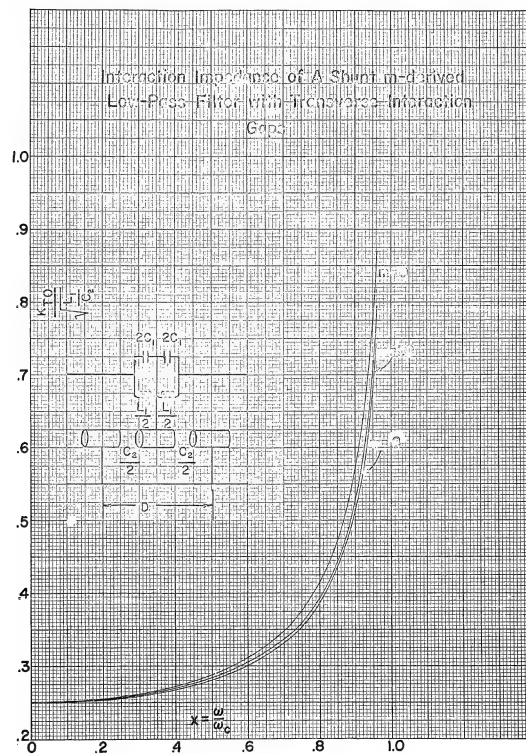
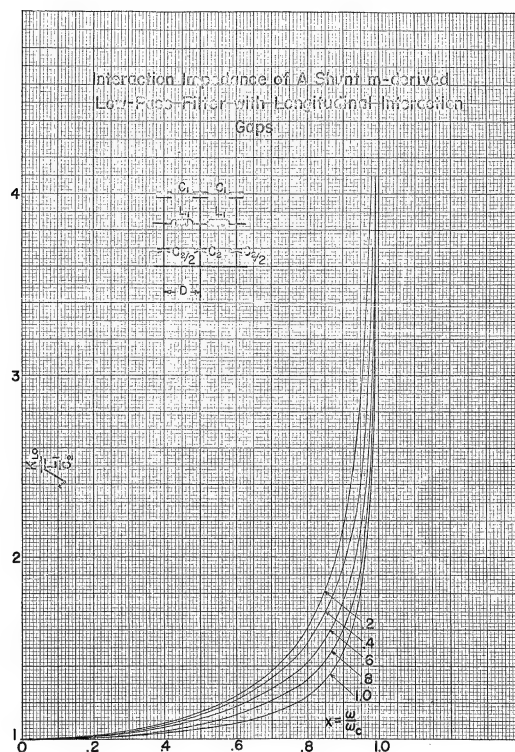
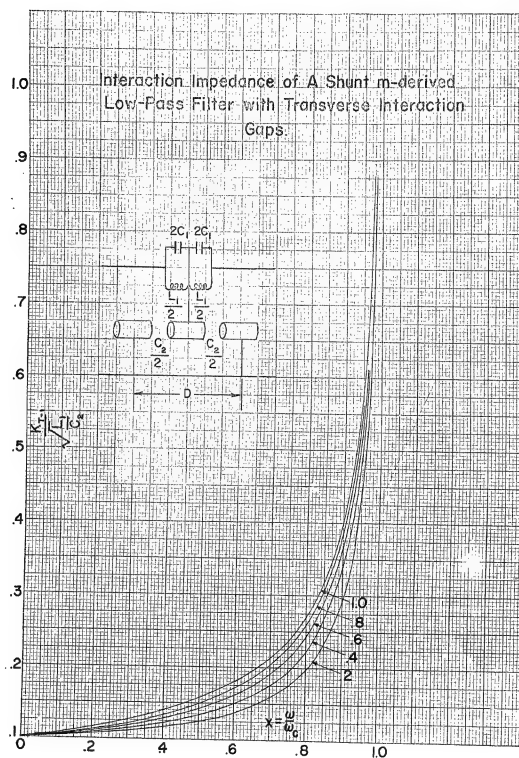


FIG. 3.6.--Transverse interaction impedance K_{T0} of a shunt m-derived low-pass circuit.

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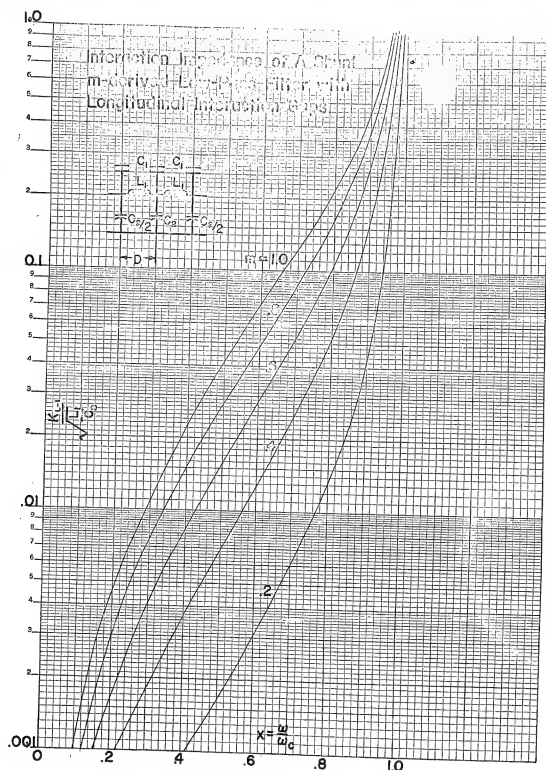


FIG. 3.12.--Longitudinal interaction impedance K_{L-1} of a shunt m-derived low-pass circuit.

IV. CHARACTERISTICS OF THE HIGH-PASS CIRCUITS

The high-pass circuits are shown in Fig. 4.1. The first two have longitudinal interaction gaps and the third one has transverse interaction gaps.

The equations for the characteristics are

$$\theta = -2\sin^{-1} \frac{m}{\sqrt{x^2 - 1 + m^2}} \quad (4.1)$$

$$|y_{pn}| = \frac{2x}{|\theta + 2n\pi|} \quad (4.2)$$

$$y_g = \frac{\sqrt{x^2 - 1}}{mx} (x^2 - 1 + m^2) \quad (4.3)$$

$$Z_{O\pi} = \sqrt{\frac{L_2}{C_1}} \left\{ \frac{x^2 - 1 + m^2}{x \sqrt{x^2 - 1}} \right\} \quad (4.4)$$

$$Z_{OT} = \sqrt{\frac{L_2}{C_1}} \left\{ \frac{\sqrt{x^2 - 1}}{x} \right\} \quad (4.5)$$

Fig. 14(a) and (c)

$$Z_{O\pi} = \sqrt{\frac{L_2}{C_1}} \left\{ \frac{x \sqrt{x^2 - 1}}{x^2 - 1 + m^2} \right\} \quad (4.6)$$

$$Z_{OT} = \sqrt{\frac{L_2}{C_1}} \left\{ \frac{x}{\sqrt{x^2 - 1}} \right\} \quad (4.7)$$

Fig. 14(b)

$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \frac{x^2 - 1 + m^2}{x \sqrt{x^2 - 1}} \quad (4.8)$$

Fig. 14(a)

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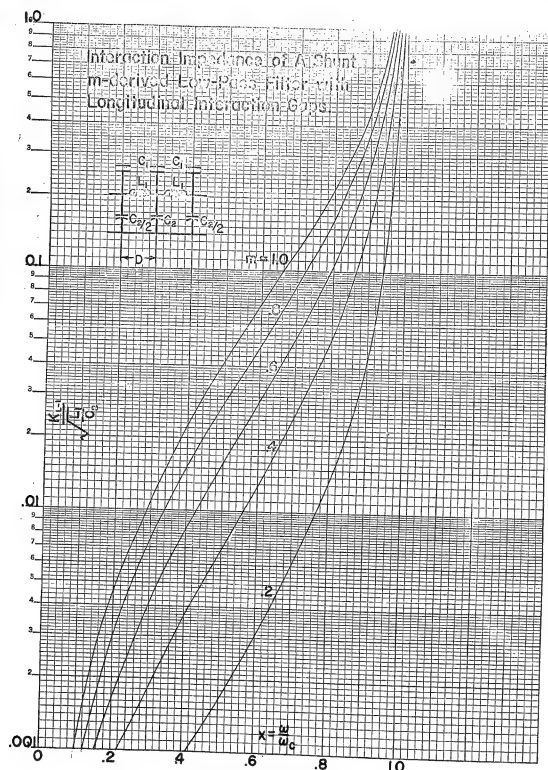


FIG. 3.12.--Longitudinal interaction impedance K_{L-1} of a shunt m-derived low-pass circuit.

IV. CHARACTERISTICS OF THE HIGH-PASS CIRCUITS

The high-pass circuits are shown in Fig. 4.1. The first two have longitudinal interaction gaps and the third one has transverse interaction gaps.

The equations for the characteristics are

$$\theta = -2\sin^{-1} \frac{m}{\sqrt{x^2-1+m^2}} \quad (4.1)$$

$$|y_{pn}| = \frac{2x}{|\theta+2n\pi|} \quad (4.2)$$

$$y_g = \frac{\sqrt{x^2-1}}{mx} (x^2-1+m^2) \quad (4.3)$$

$$Z_{O\pi} = \sqrt{\frac{L_2}{C_1}} \frac{x^2-1+m^2}{x\sqrt{x^2-1}} \quad (4.4)$$

$$Z_{OT} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{x^2-1}}{x} \quad (4.5)$$

Fig. 14(a) and (c)

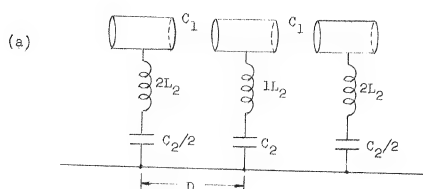
$$Z_{OT} = \sqrt{\frac{L_2}{C_1}} \frac{x\sqrt{x^2-1}}{x^2-1+m^2} \quad (4.6)$$

$$Z_{O\pi} = \sqrt{\frac{L_2}{C_1}} \frac{x}{\sqrt{x^2-1}} \quad (4.7)$$

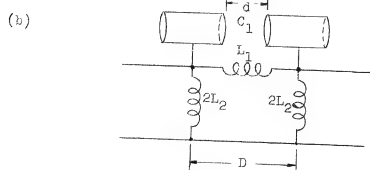
Fig. 14(b)

$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \frac{x^2-1+m^2}{x\sqrt{x^2-1}} \quad (4.8)$$

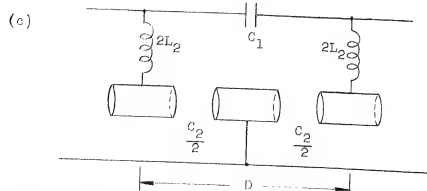
Fig. 14(a)



Series m-derived circuit with longitudinal interaction gaps.



Shunt m-derived circuit with longitudinal interaction gaps.



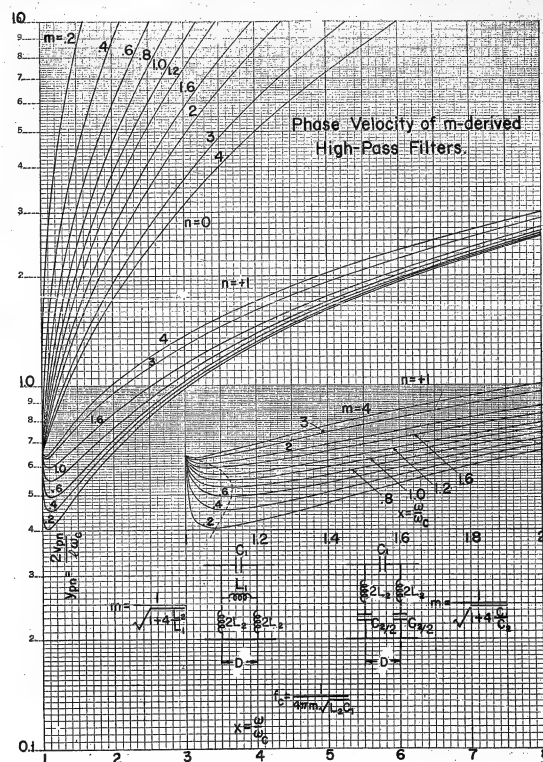
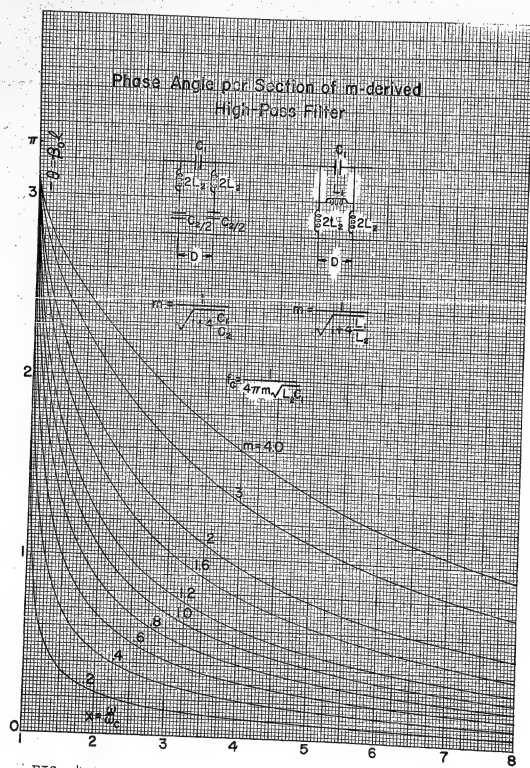
Series m-derived circuit with transverse interaction gaps.

FIG. 4.1.--High-pass circuits.

$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \frac{x}{\sqrt{x^2 - 1}} \quad \text{Fig. 14(b)} \quad (4.9)$$

$$K_{Tn} = \sqrt{\frac{L_2}{C_1}} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \frac{x^2 - 1 + m^2}{x \sqrt{x^2 - 1}} \quad \text{Fig. 14(c)} \quad (4.10)$$

These equations are plotted as curves in Figs. 4.2 to 4.12.



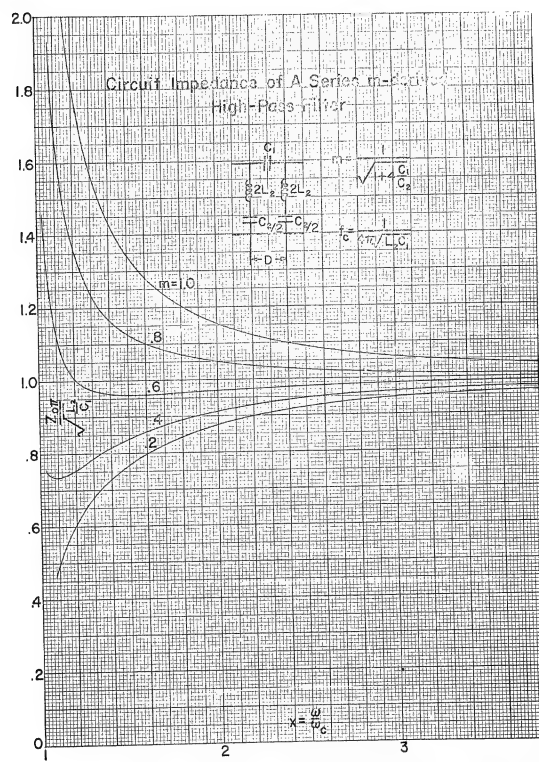
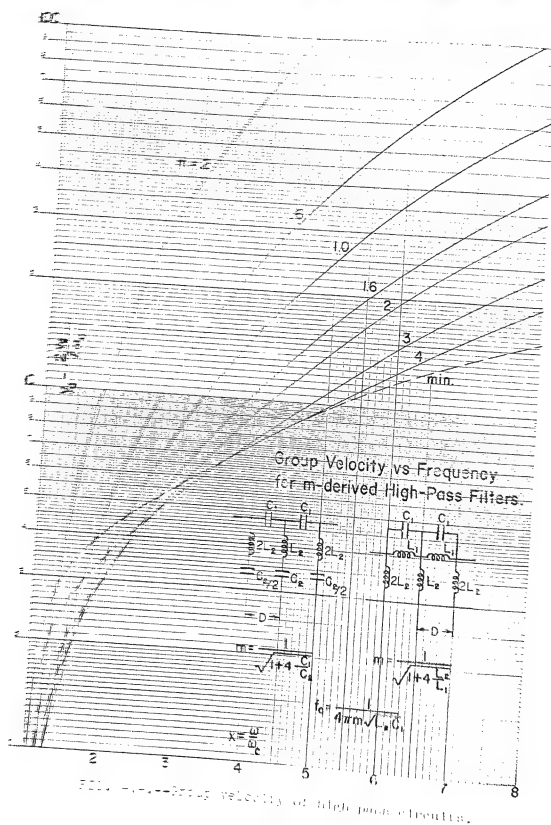


FIG. 4.5.--Circuit impedance of high-pass circuits of Pi section.

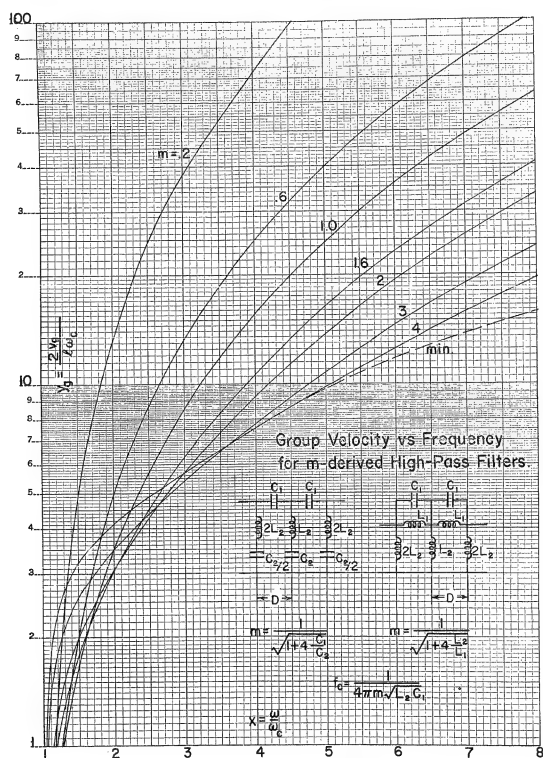


FIG. 4.4.--Group velocity of high-pass circuits.

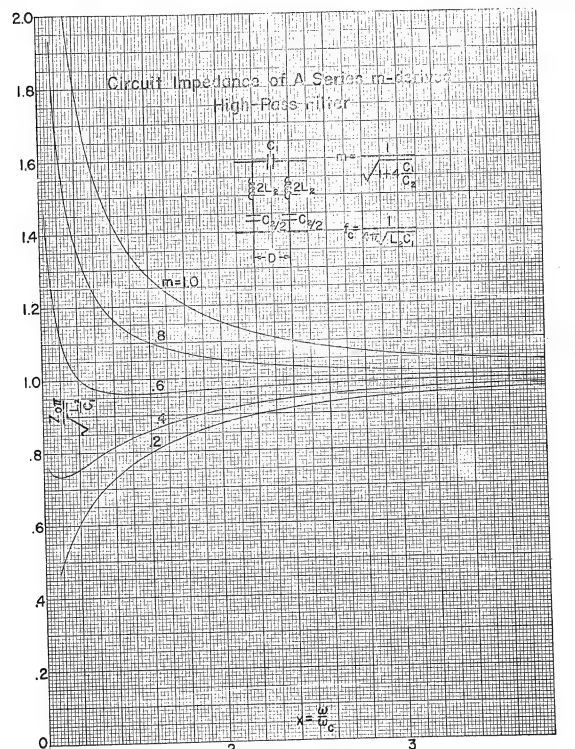


FIG. 4.5.--Circuit impedance of high-pass circuits of Pi section.

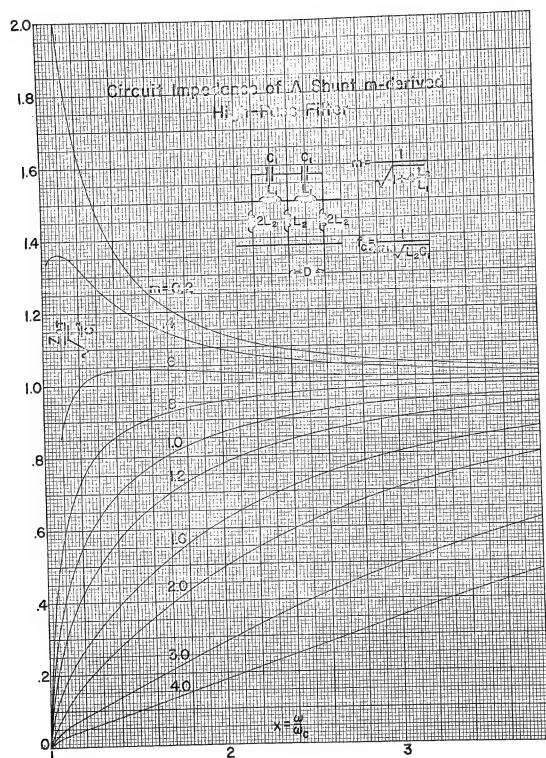


FIG. 4.6.--Circuit impedance of high-pass circuits of Tee section.

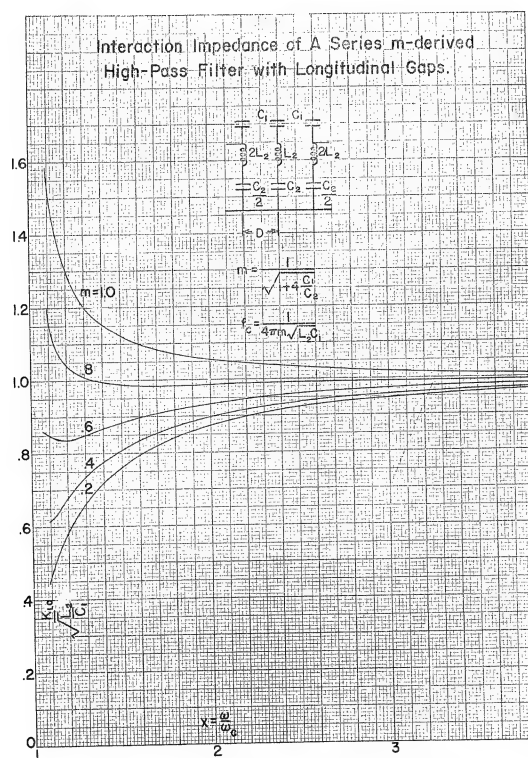


FIG. 4.7.--Longitudinal interaction impedance K_{LO} of a series m-derived high-pass circuit.

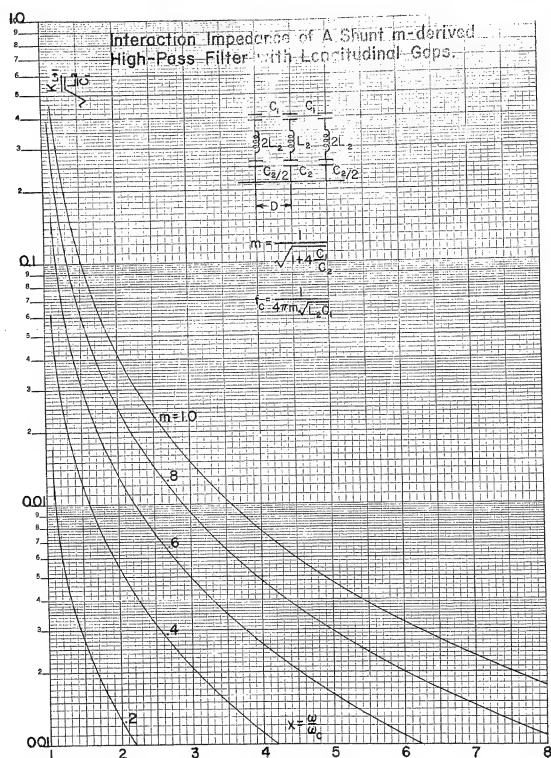


FIG. 4.8.--Longitudinal interaction impedance K_{L+1} of a series m -derived high-pass circuit.

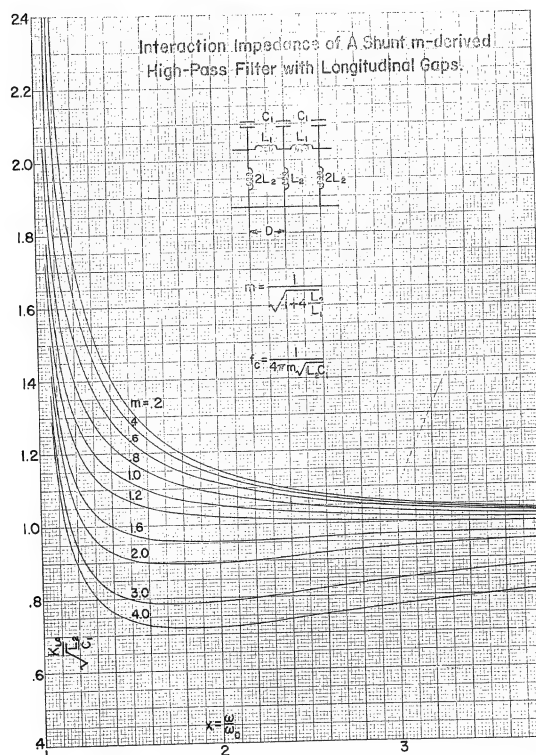


FIG. 4.9.--Longitudinal interaction impedance K_{L0} of a shunt m -derived high-pass circuit.

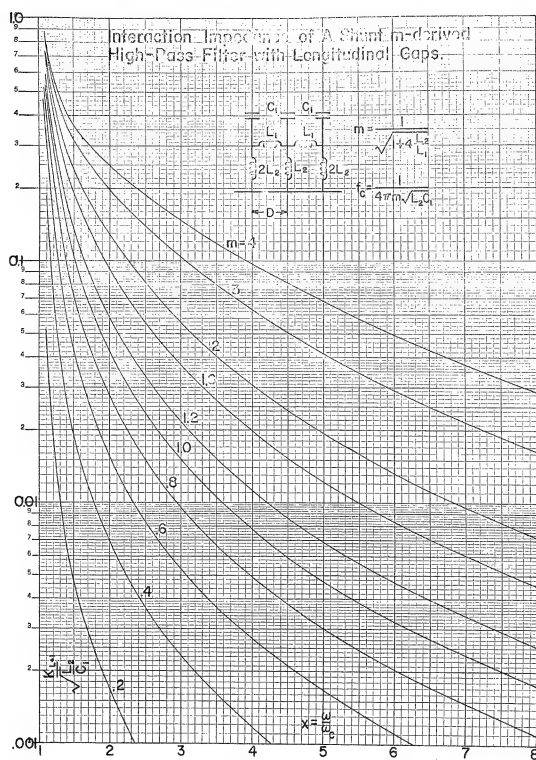


FIG. 4.10.--Longitudinal interaction impedance K_{L+1} of a shunt m-derived high-pass circuit.

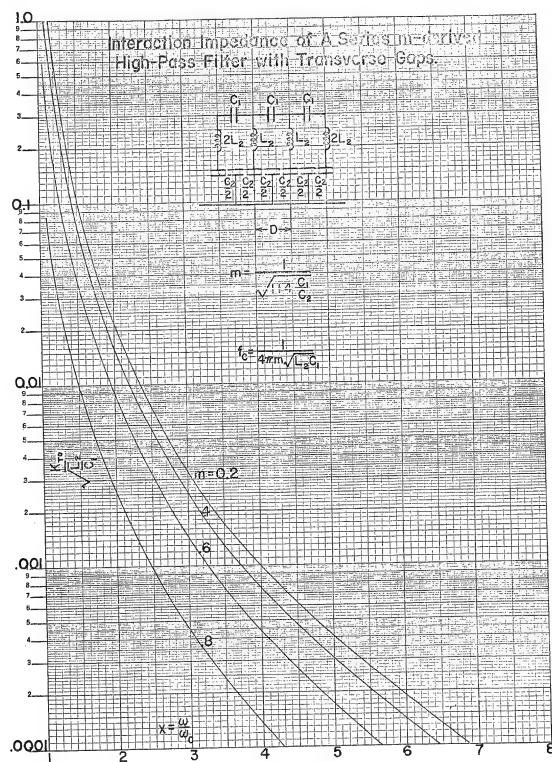


FIG. 4.11.--Transverse interaction impedance K_{T0} of a series m-derived high-pass circuit.

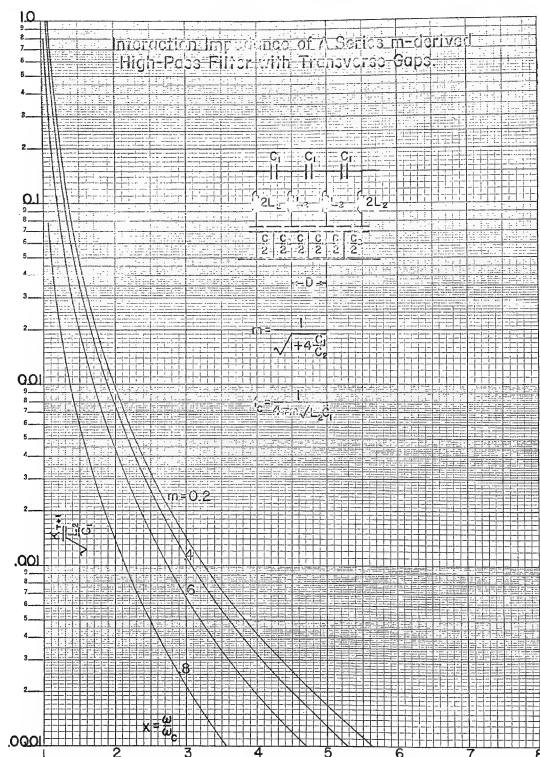


FIG. 4.12.--Transverse interaction impedance K_{T+1} of a series m-derived high-pass circuit.

V. CHARACTERISTICS OF THE THREE-ELEMENT INDUCTIVELY COUPLED BAND-PASS CIRCUIT

The inductively coupled band-pass circuit is shown in Fig. 5.1.

The equations for the characteristics are

$$\theta = \pm 2 \sin^{-1} \sqrt{\frac{x^2 - m^2}{1 - m^2}} \quad (5.1)$$

$$y_{pn} = \frac{2x}{\theta + 2n\pi} \quad (5.2)$$

$$y_g = \frac{\sqrt{1-x^2} \sqrt{x^2 - m^2}}{x} \quad (5.3)$$

$$Z_{OT} = \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-m^2}}{\sqrt{x^2 - m^2}} \frac{x}{\sqrt{1-x^2}} \quad (5.4)$$

$$Z_{OT} = \sqrt{\frac{L_1}{C_2}} \frac{x \sqrt{x^2 - 1}}{\sqrt{m^2 - x^2} \sqrt{1 - m^2}} \quad (5.5)$$

$$K_{Tn} = \left(\frac{\sin \frac{\theta}{2}}{\frac{\theta}{2}} \right)^2 \frac{1}{4} \sqrt{\frac{L_1}{C_2}} \frac{\sqrt{1-m^2}}{\sqrt{x^2 - m^2}} \frac{x}{\sqrt{1-x^2}} \quad (5.6)$$

The cutoff frequencies are given by $x_c = 1$ or $\omega_c = 1/m_s \sqrt{L_2 C_2}$ and $x_1 = m_s$ where $m_s = 1/\sqrt{1+4L_2/L_1}$

For positive mutual coupling, i.e. when L_2 and L_1 are both greater than zero, the circuit is fundamentally a forward-wave circuit and the normalized pass-band is between m_s and 1.

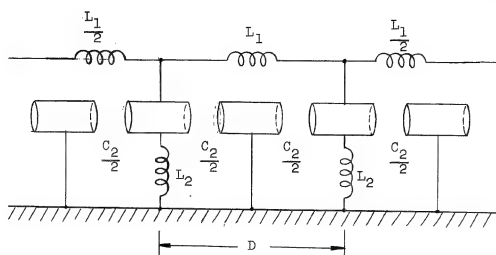


FIG. 5.1.--Three-element inductively coupled band-pass circuit.

When L_2 or L_1 is negative, the circuit is fundamentally a backward-wave circuit and the normalized pass-band is between 1 and m_s . For the fundamental forward-wave circuit, the positive sign is to be used in the θ expression and for the fundamental backward-wave circuit the negative sign is to be used in the θ expression. The negative-mutual-coupled circuit requires a center-tapped coil with the two halves wound in the same direction. The positive-mutual-coupled circuit on the other hand requires a center-tapped coil with the two halves wound in the opposite direction. A band-pass space-harmonic amplifier can be designed using the negative-mutual-coupled circuit with good bandwidth when the pass band is between about 1 to 1.60.

Equations (4.1) to (4.6) are plotted in Figs. 5.2 to 5.16. Cases for $m_s = 1.5, 1.6$ and 1.7 are also plotted on an enlarged scale. Curves include both the positive and the negative-mutual-coupling cases.

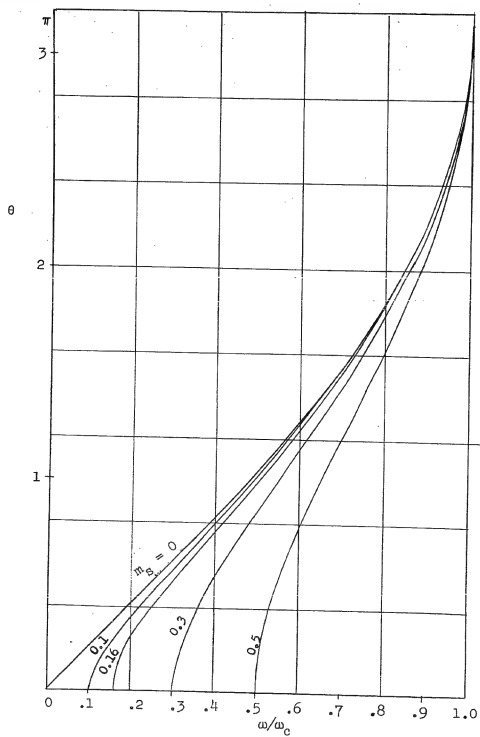


FIG. 5.2.--Phase angle per section of the positive mutual coupled three-element band-pass circuit.

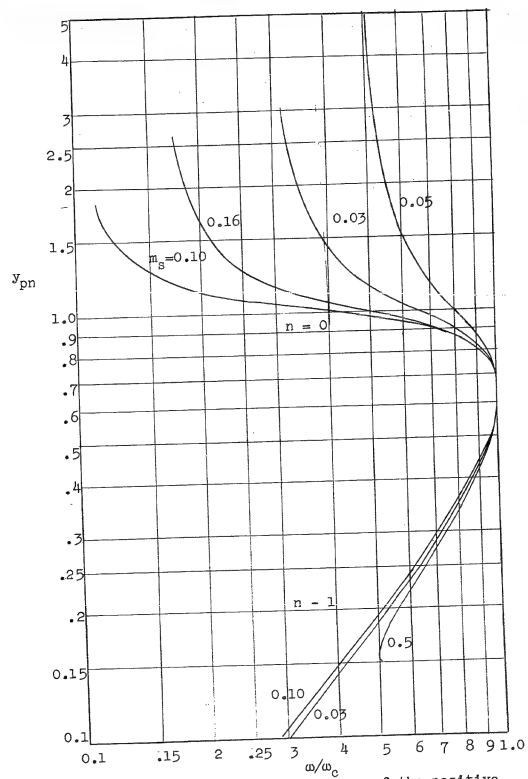


FIG. 5.3.--Phase velocity y_{pn} of the positive-mutual-coupled three-element band-pass circuit.

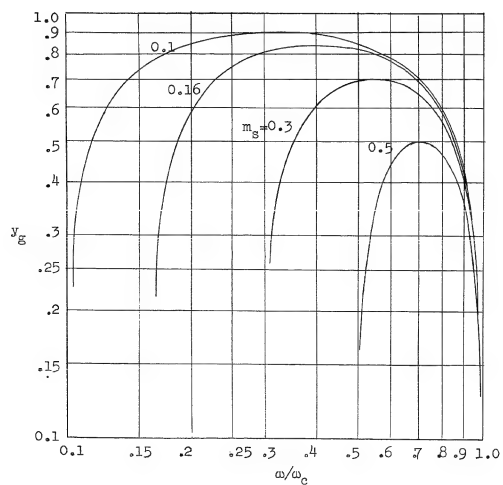


FIG. 5.4.--Group velocity y_g of the positive-mutual-coupled three-element band-pass circuit.

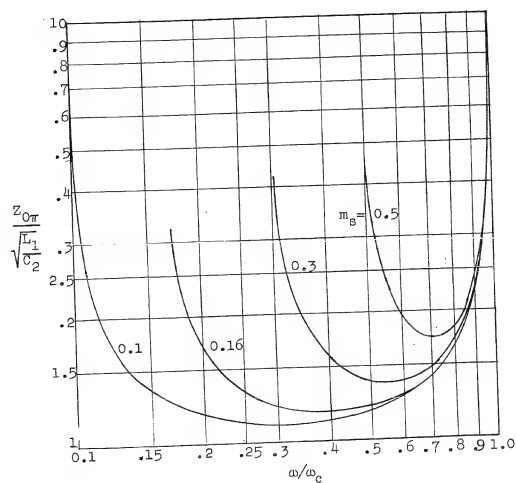


FIG. 5.5.--Circuit impedance Z_{OT} of the positive-mutual-coupled three-element band-pass circuit of Pi section.

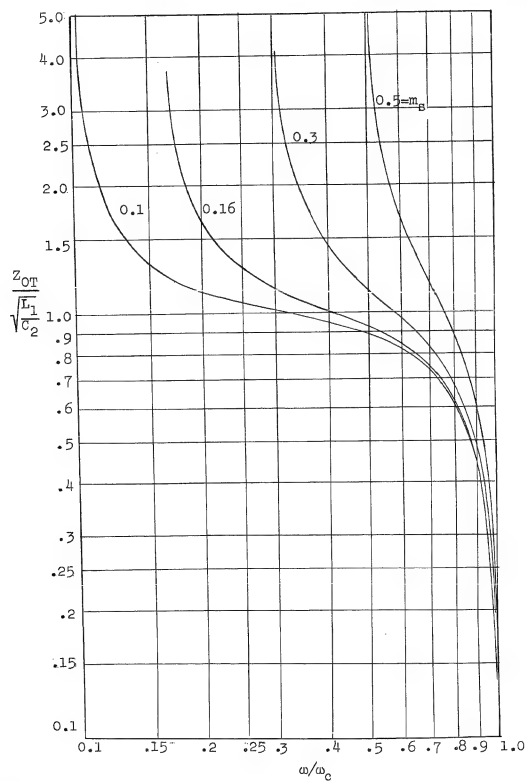


FIG. 5.6.--Circuit impedance Z_{OT} of the positive-mutual coupled three-element band-pass circuit of Tee section.

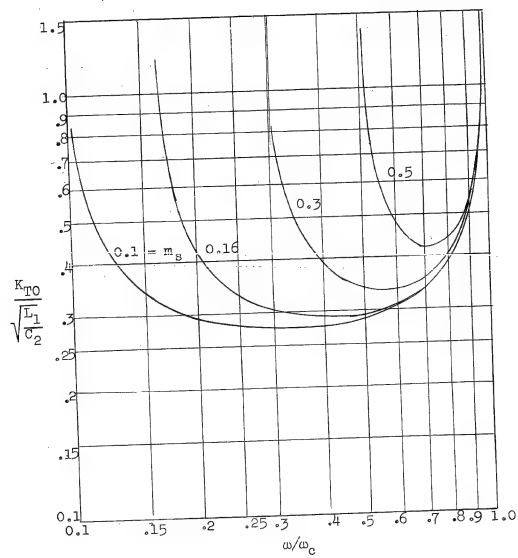


FIG. 5.7.--Transverse interaction impedance K_{T0} of the positive-mutual-coupled three-element band-pass circuit.

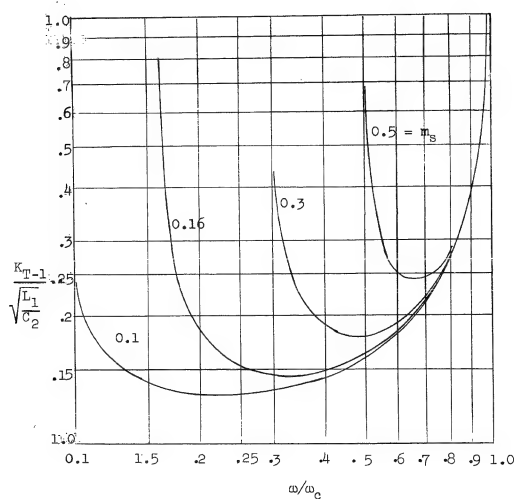


FIG. 5.8.--Transverse interaction impedance K_{T-1} of the positive-mutual-coupled three-element band-pass circuit.

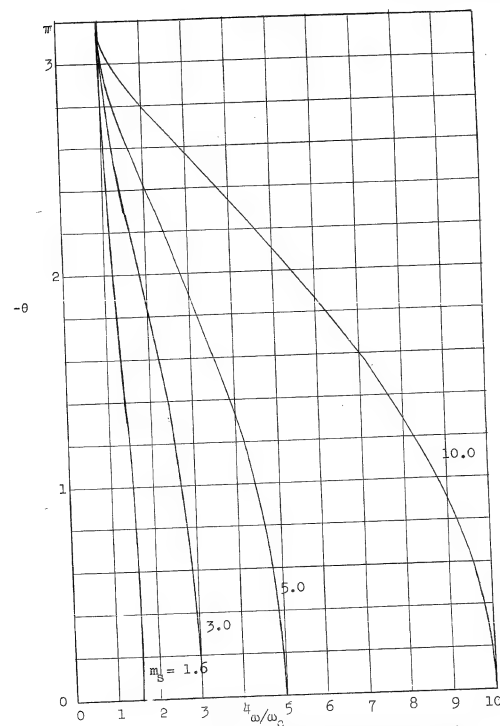


FIG. 5.9.--Phase angle per section of the negative-mutual-coupled three-element band-pass circuit.

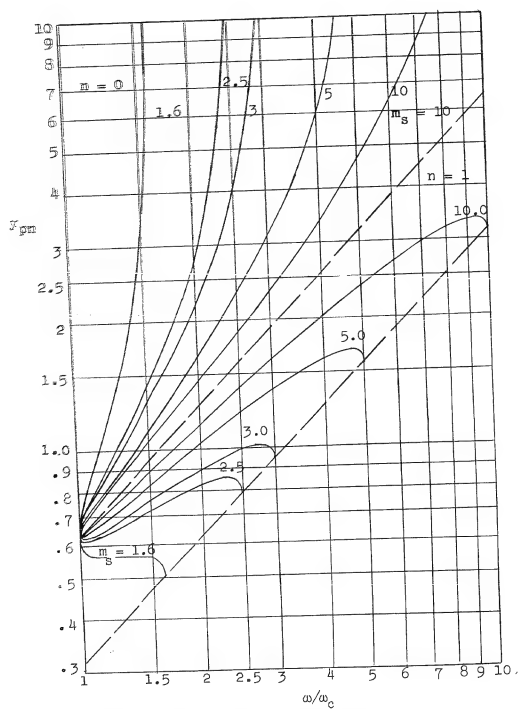


FIG. 5.10.--Phase velocity y_{pb} of the negative-mutual-coupled three-element band-pass circuit.

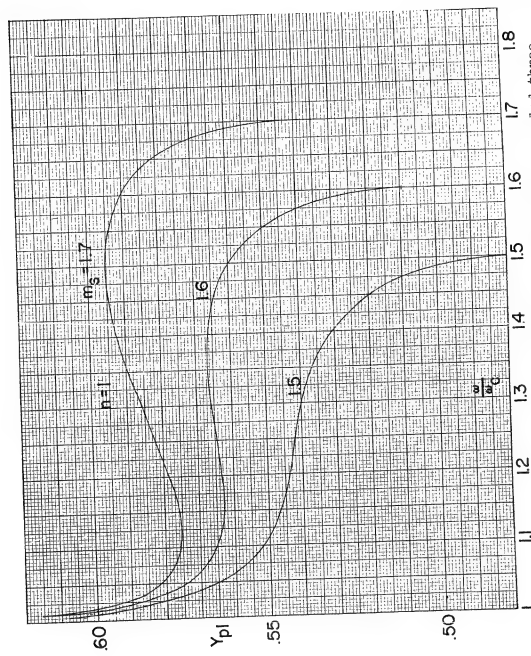


FIG. 5.11.--Phase velocity of the negative-mutual-coupled three-element band-pass circuit (enlarged scale).

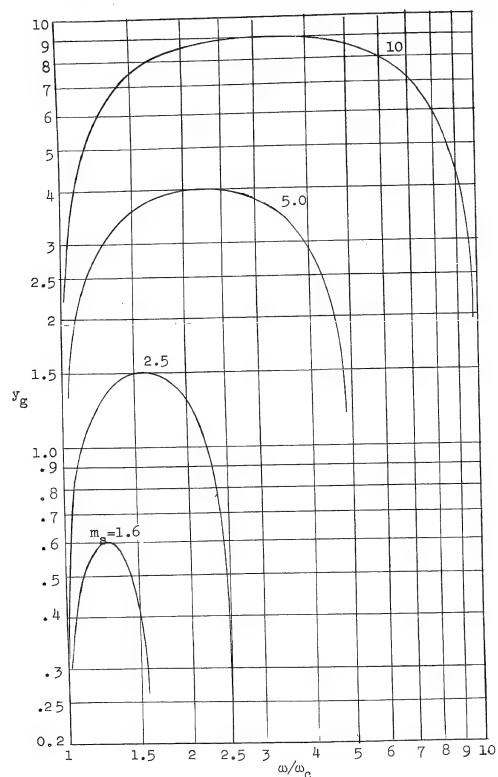


FIG. 5.12.--Group velocity y_g of the negative-mutual-coupled three-element band-pass circuit.

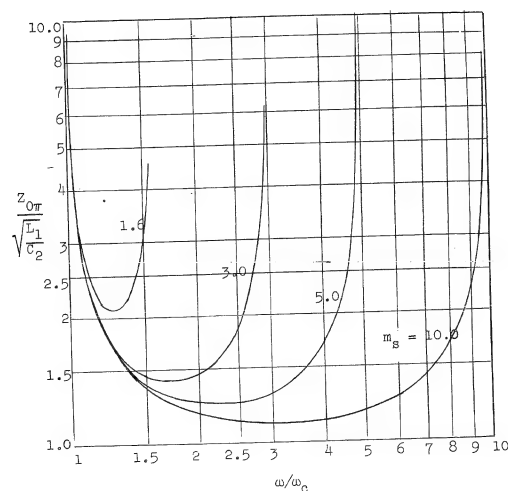


FIG. 5.13.--Circuit impedance Z_{0T} of the negative-mutual-coupled three-element band-pass circuit of P1 section.

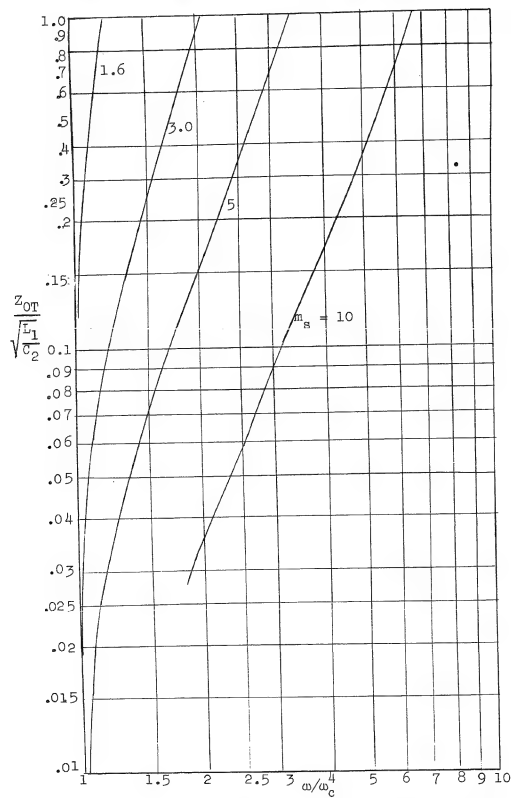


FIG. 5.14.--Circuit impedance Z_{OT} of the negative-mutual-coupled three-element band-pass circuit of Tee section.

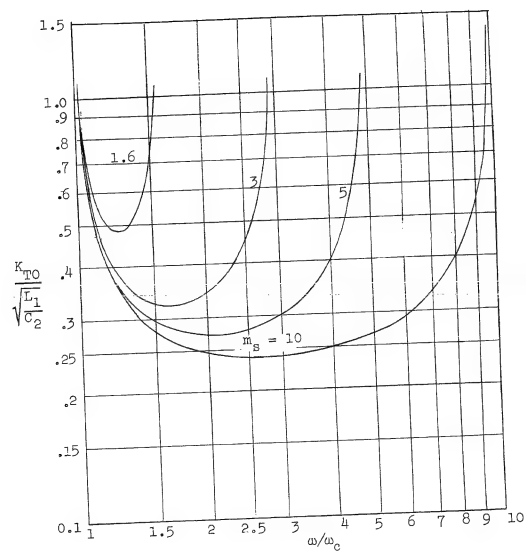


FIG. 5.15.--Transverse interaction impedance K_{T0} of the negative-mutual-coupled three-element band-pass circuit.

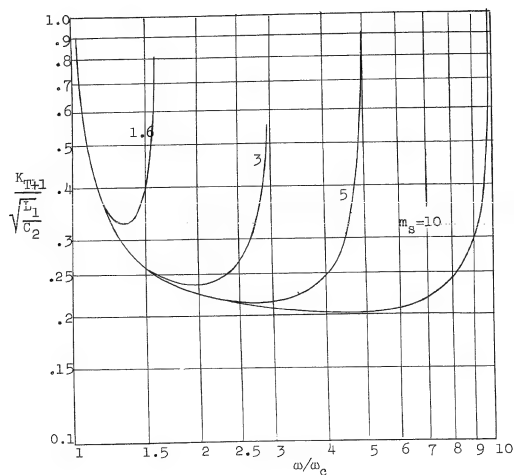


FIG. 5.16.--Transverse interaction impedance K_{Tn1} of the negative-mutual-coupled three-element band-pass circuit.

VI. CHARACTERISTICS OF THE THREE-ELEMENT CAPACITIVELY COUPLED BAND-PASS CIRCUITS

The capacitively coupled band-pass circuits are shown in Figs. 6.1 (a) and (b).

The equations for the characteristics are

$$\theta = -2\sin^{-1} \sqrt{\frac{x^2 - m_r^2}{1 - m_r^2}} \quad (6.1)$$

$$y_{pn} = \frac{2x}{|\theta + 2\pi n|} \quad (6.2)$$

$$y_g = x^2 \sqrt{x^2 - 1} \sqrt{x^2 - m_r^2} \quad (6.3)$$

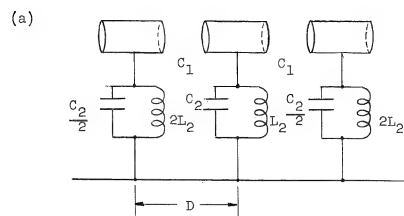
$$Z_{O\pi} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \quad (6.4)$$

$$Z_{OT} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - x^2}}{\sqrt{1 - m_r^2} \sqrt{x^2 - m_r^2} x} \quad (6.5)$$

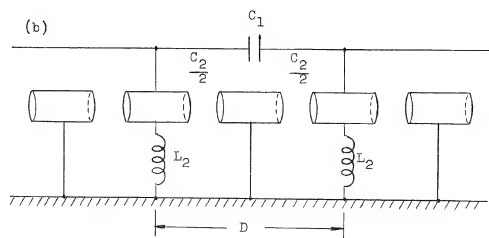
$$K_{Ln} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (6.6)$$

$$K_{Tn} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{1 - m_r^2}}{x \sqrt{1 - x^2} \sqrt{x^2 - m_r^2}} \frac{1}{4} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (6.7)$$

$$\text{and } \omega_0 = \frac{m_r}{\sqrt{L_2 C_2}} \quad \text{where } m_r = x_1^{-1} = \sqrt{\frac{1}{1 + \frac{4C_1}{C_2}}}$$



Longitudinal interaction structure



Transverse interaction structure

FIG. 6.1.--Three-element capacitively coupled band-pass circuits.

The capacitively coupled circuit is a backward-wave circuit. There are two possible locations for the interaction electrodes, i.e., at C_1 for the longitudinal type and at C_2 for the transverse type.

Equations (6.1) to (6.7) are plotted in Figs. 6.2 to 6.13.

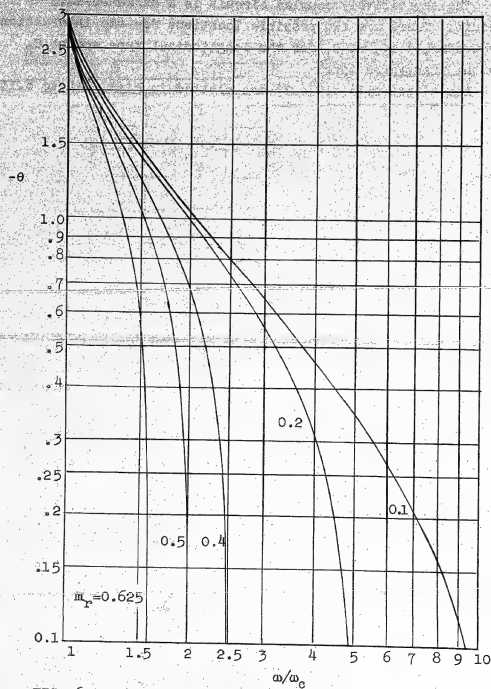


FIG. 6.2.--Phase angle per section of the three-element capacitively coupled band-pass circuit.

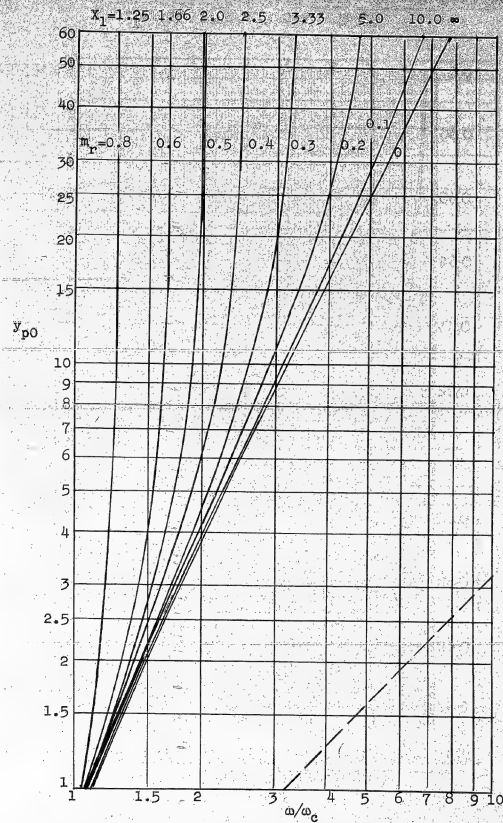


FIG. 6.3.--Phase velocity v_{p0} of the three-element capacitively coupled band-pass circuit.

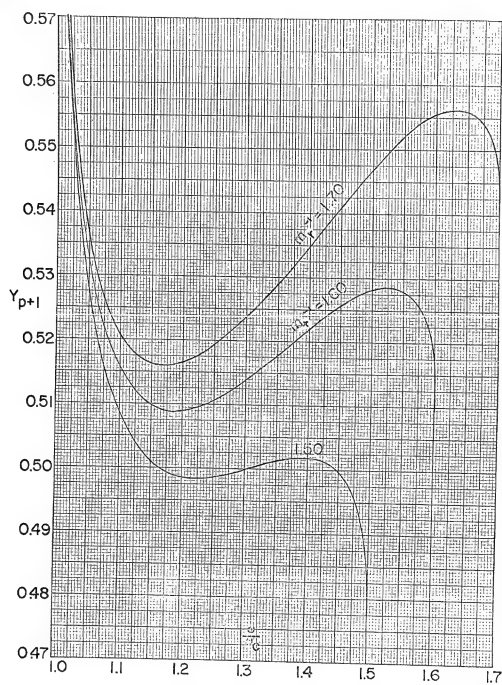


FIG. 6.4.--Phase velocity of the three-element capacitively coupled band-pass circuit (enlarged scale) Y_{p+1} .

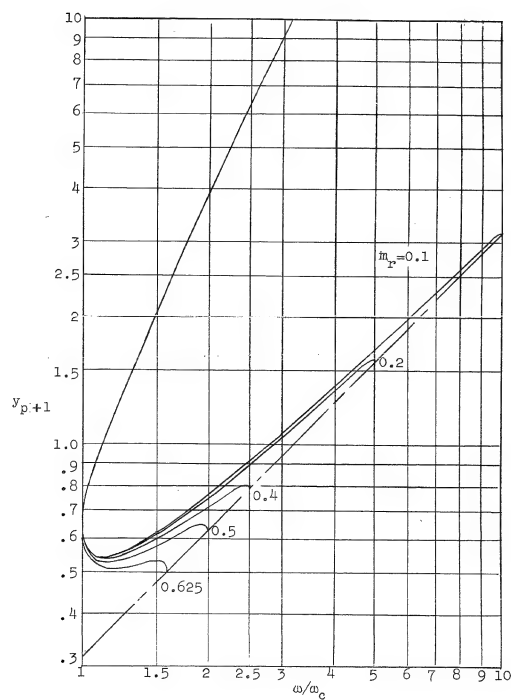


FIG. 6.5.--Phase velocity Y_{p+1} of the three-element capacitively coupled band-pass circuit.

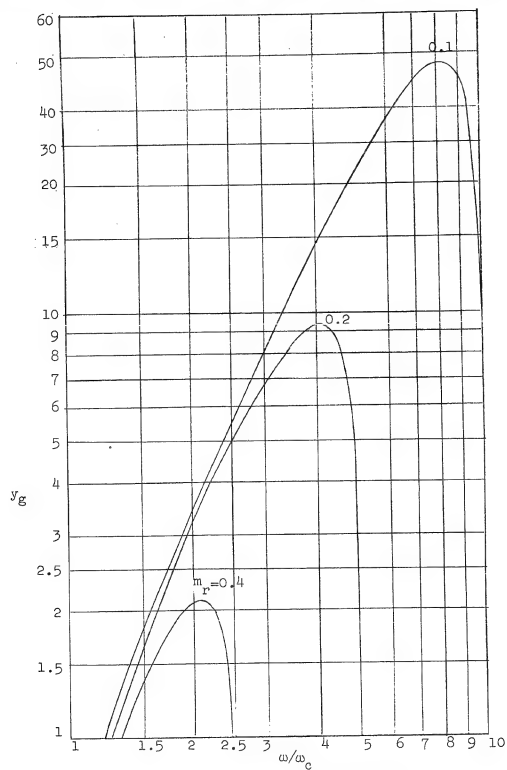


FIG. 6.6.--Group velocity y_g of the three-element capacitively coupled band-pass circuit.

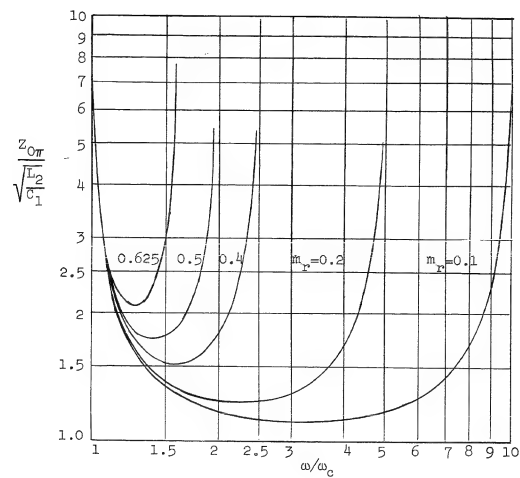


FIG. 6.7.--Circuit impedance $Z_{0\pi}$ of the three element capacitively coupled band-pass circuit of P1 section.

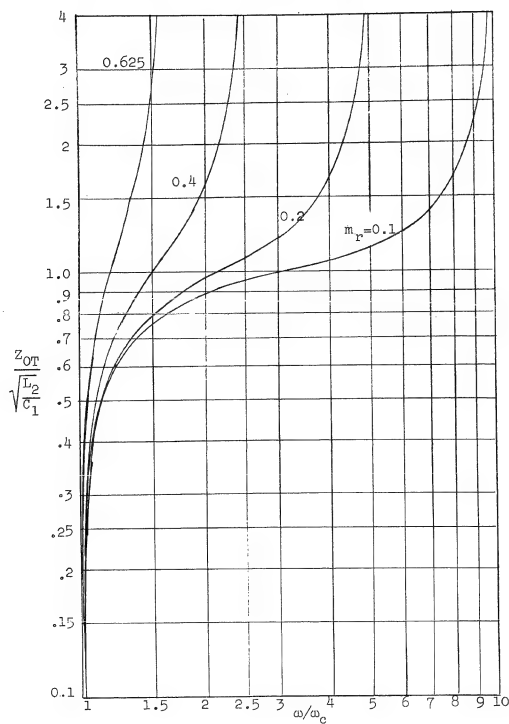


FIG. 6.8.--Circuit impedance Z_{OT} of the three-element capacitively coupled band-pass circuit of Tee section.

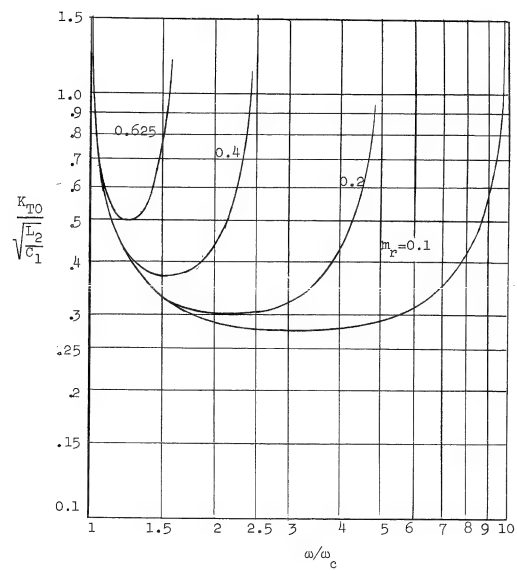


FIG. 6.9.--Transverse interaction impedance K_{T0} of the three-element capacitively-coupled band-pass circuit.

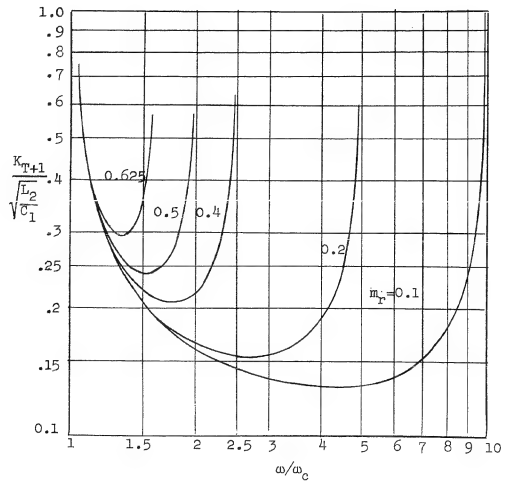


FIG. 6.10.--Transverse interaction impedance K_{T+1} of the three-element capacitively-coupled band-pass circuit.

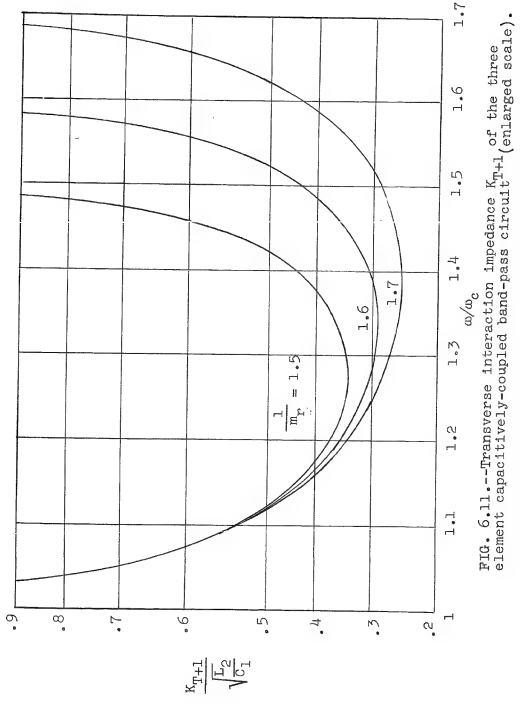


FIG. 6.11.--Transverse interaction impedance K_{T+1} of the three element capacitively-coupled band-pass circuit¹⁴ (enlarged scale).

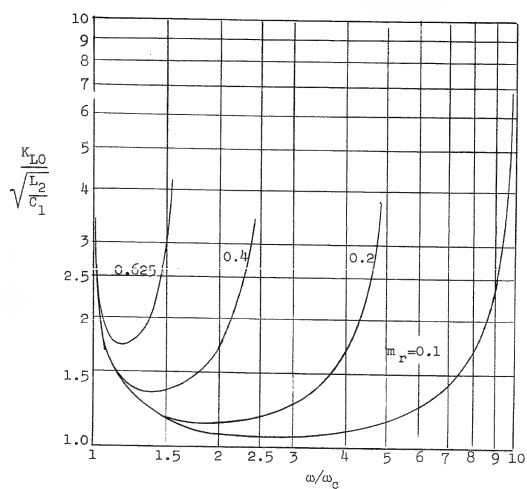


FIG. 6.12.--Longitudinal interaction impedance K_{L0} of the three-element capacitively-coupled band-pass circuit.

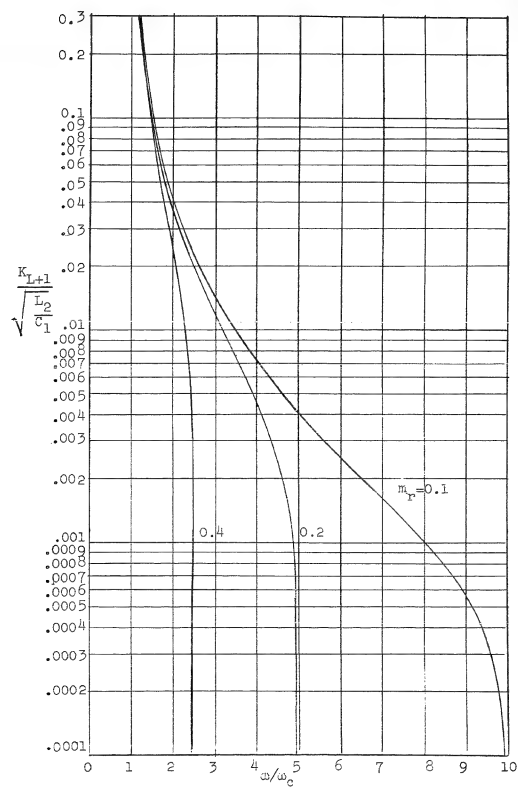


FIG. 6.13.--Longitudinal interaction impedance K_{L+1} of the three-element capacitively-coupled band-pass circuit.

VII. CHARACTERISTICS OF THE FOUR-ELEMENT BAND-PASS CIRCUIT

The four-element band-pass circuits are shown in Fig. 7.1. The equations for the characteristics are

$$\theta = \pm 2s \sin^{-1} \sqrt{\frac{x^2 - x_1^2}{(1 - x_1^2)^2 + r(1 - x^2)}} \quad (7.1)$$

$$y_{pn} = \frac{2x}{|\theta + 2n\pi|} \quad (7.2)$$

$$y_g = \frac{\sqrt{1 - x^2} \sqrt{x^2 - x_1^2} \left(1 + r \frac{1 - x^2}{1 - x_1^2}\right)}{x \sqrt{1 + r}} \quad (7.3)$$

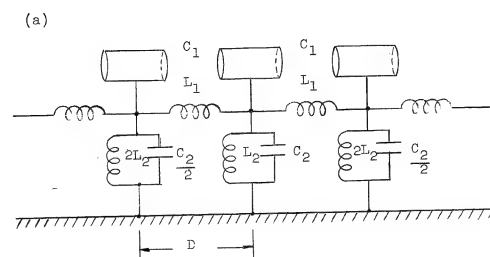
$$z_{Or} = \sqrt{\frac{L_2}{C_1}} \frac{\sqrt{r} x}{\sqrt{x^2 - x_1^2} \sqrt{1 + s} \sqrt{1 - x^2}} \quad (7.4)$$

$$z_{OT} = \sqrt{\frac{L_2}{C_1}} \sqrt{\frac{r}{s}} \frac{x}{x_1} \sqrt{\frac{(s+1) - (r+1) \frac{x^2}{x_1^2}}{s \left(\frac{x^2}{x_1^2} - 1\right) \left(1 - \frac{r}{s} \frac{x^2}{x_1^2}\right)^2}} \quad (7.5)$$

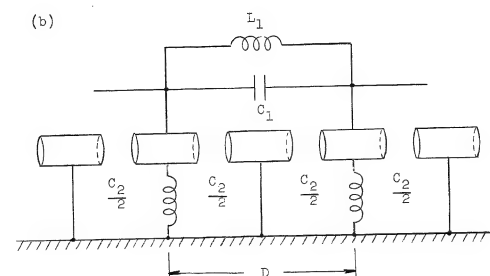
$$K_{Ln} = z_{Or} \left(\frac{\sin \frac{\beta_n D}{2}}{\frac{\beta_n D}{2}} \right)^2 \quad (7.6)$$

$$K_{Tn} = z_{Or} \frac{1}{4} \left(\frac{\sin \frac{\beta_n D}{4}}{\frac{\beta_n D}{4}} \right)^2 \quad (7.7)$$

$$\omega_c = \frac{1}{\sqrt{L_2 C_2}} \frac{\sqrt{1+s}}{\sqrt{1+r}}, \quad x_1 = \frac{m_s}{m_r} \quad (7.8)$$



Longitudinal interaction structure.



Transverse interaction structure.

FIG. 7.1.--Four-element band-pass circuits.

where

$$m_r = \frac{1}{\sqrt{1+r}} = \frac{1}{\sqrt{1+\frac{4C_1}{C_2}}}$$

$$m_s = \frac{1}{\sqrt{1+s}} = \frac{1}{\sqrt{1+\frac{4L_2}{L_1}}}$$

For the fundamental-backward-wave circuit, x_1 is greater than unity and the negative sign in the θ expression should be used. For the fundamental-forward-wave circuit, x_1 is less than unity and the positive sign in the θ expression should be used.

For the case of negative mutual coupling, L_1 is negative and L_2 is positive. The circuit is then fundamentally a backward-wave circuit. When L_1 and L_2 are both positive, i.e. for the case of positive mutual coupling, the circuit can fundamentally be a forward-wave or a backward-wave circuit depending on whether x_1 is less than unity or greater than unity.

Curves are presented in Figs. 7.2 to 7.91 for both the forward-wave and the backward-wave circuits.

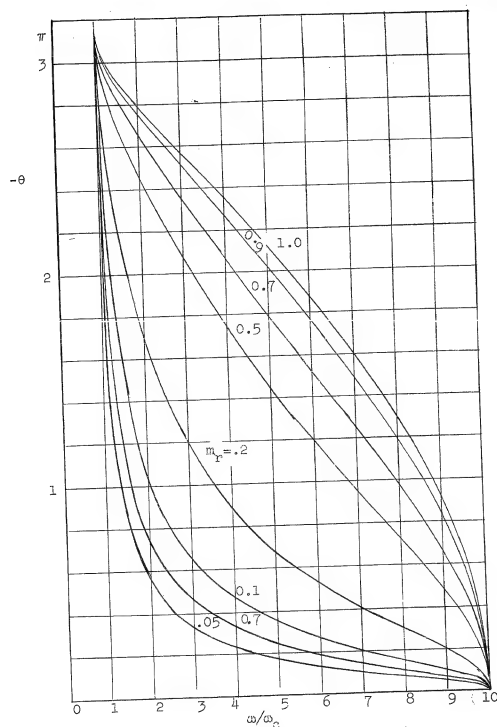


FIG. 7.2.--Phase angle per section of the four-element band-pass circuit, $x_1=5$.

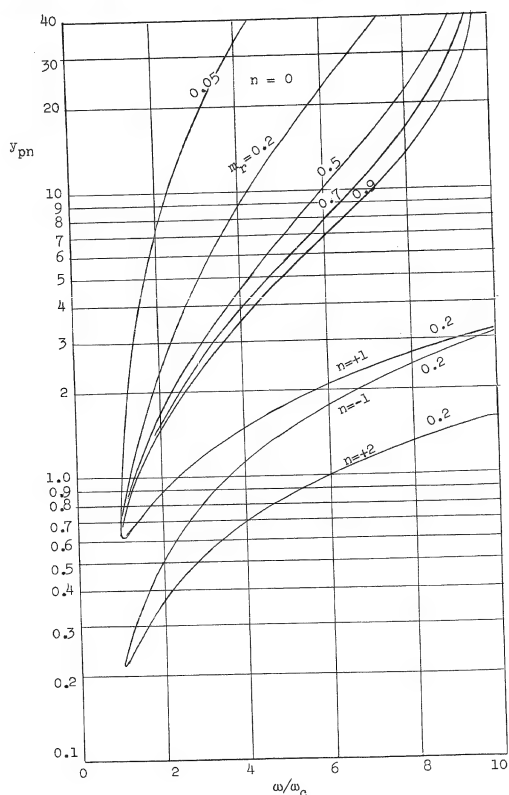


FIG. 7.3.--Phase velocity v_{ph} of the four-element band-pass circuit, $x_1=10$.

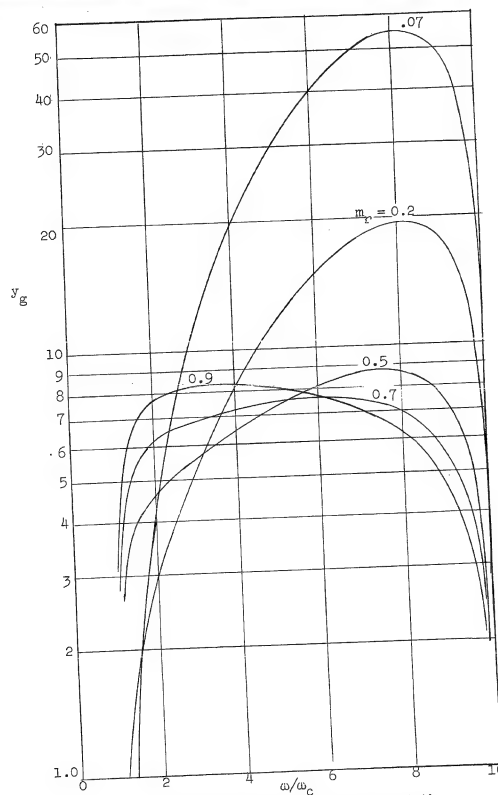


FIG. 7.4.--Group velocity v_g of the four-element band-pass circuit, $x_1=10$.

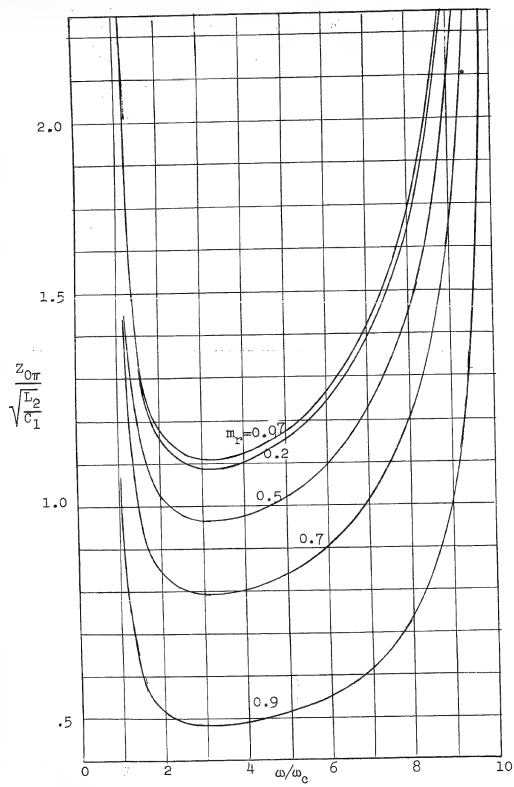


FIG. 7.5.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Pi section, $x_1 = 10$.

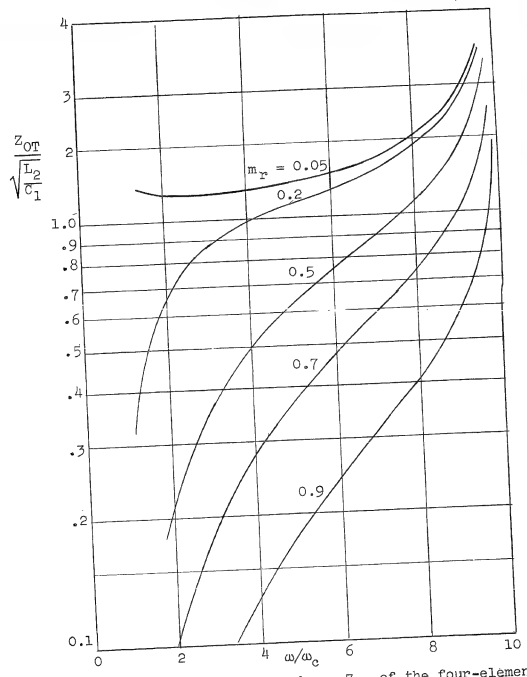
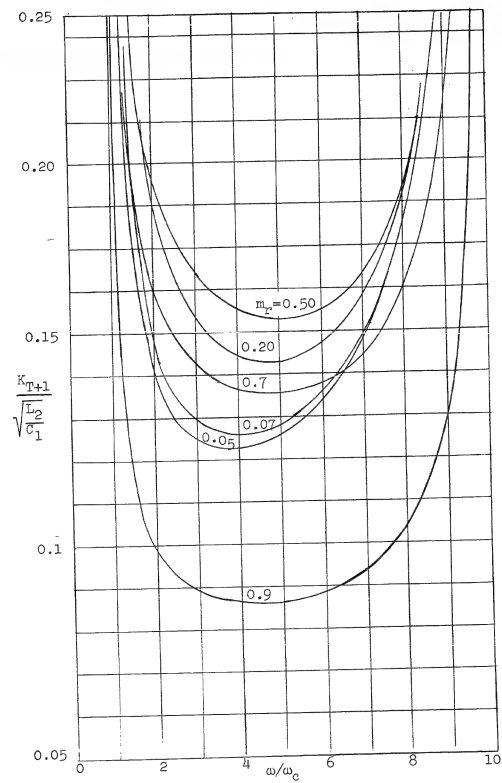
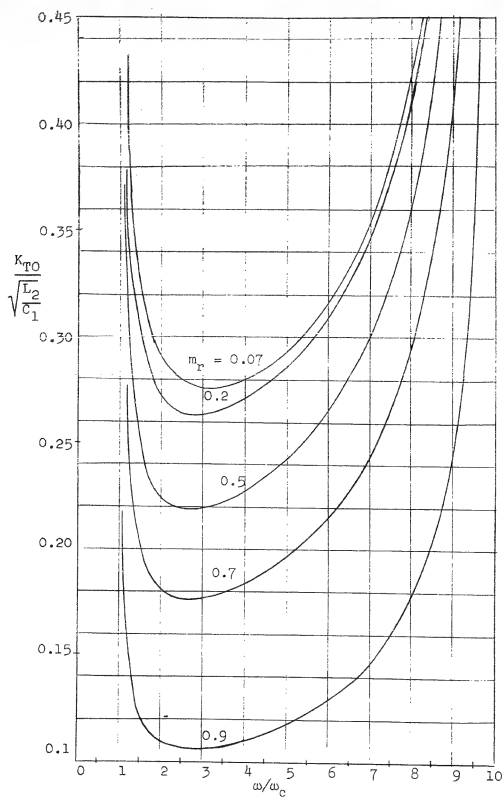


FIG. 7.6.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $x_1 = 10$.



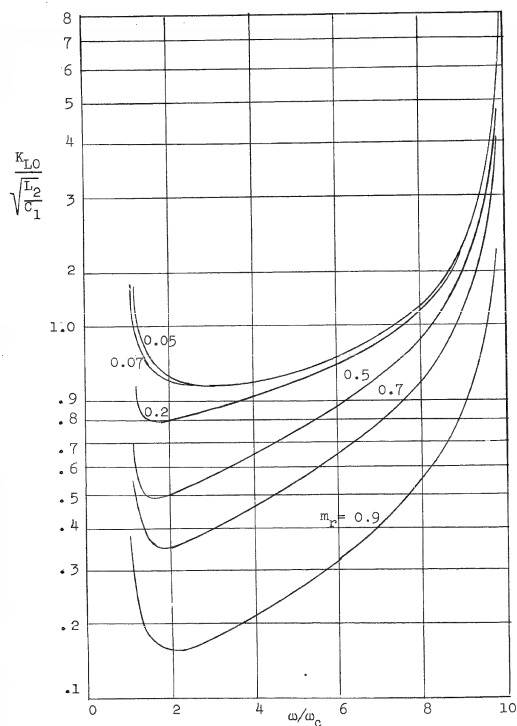


FIG. 7.9.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 10$.

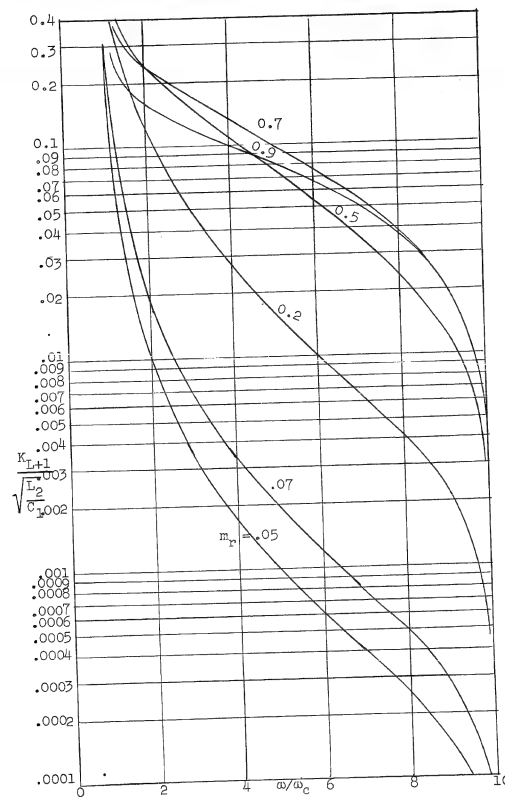


FIG. 7.10.--Longitudinal interaction impedance K_{L+1} of the four-element band-pass circuit, $x_1 = 10$.

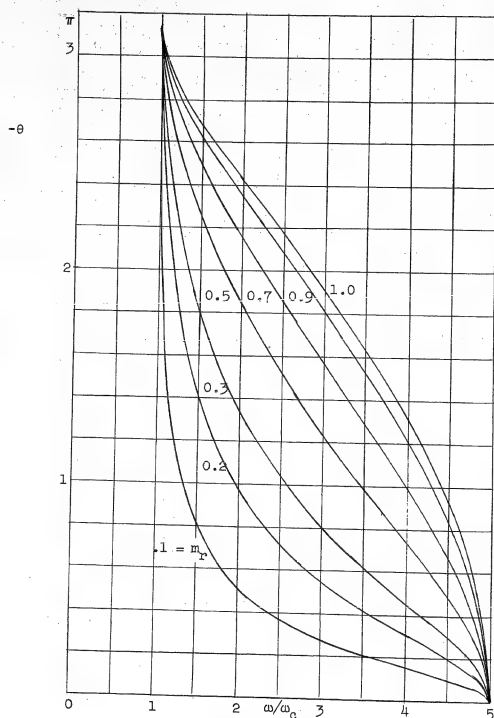


FIG. 7.11.--Phase angle per section of the four-element band-pass circuit, $x_1 = 5$.

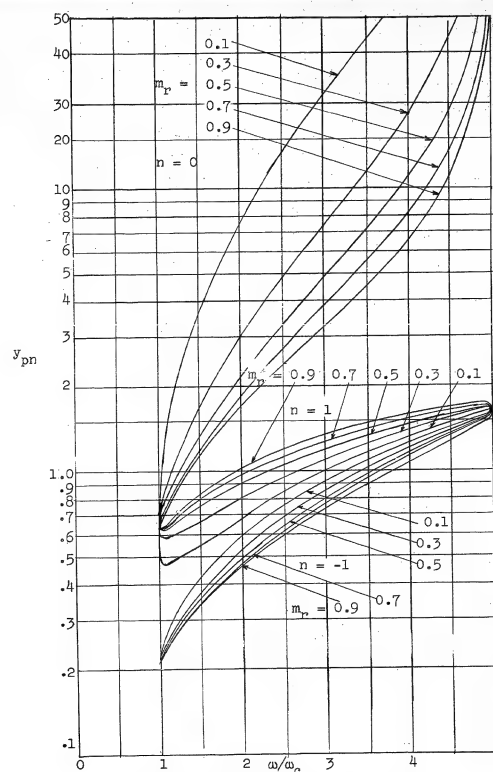


FIG. 7.12.--Phase velocity y_{pn} of the four-element band-pass circuit, $x_1 = 5$.

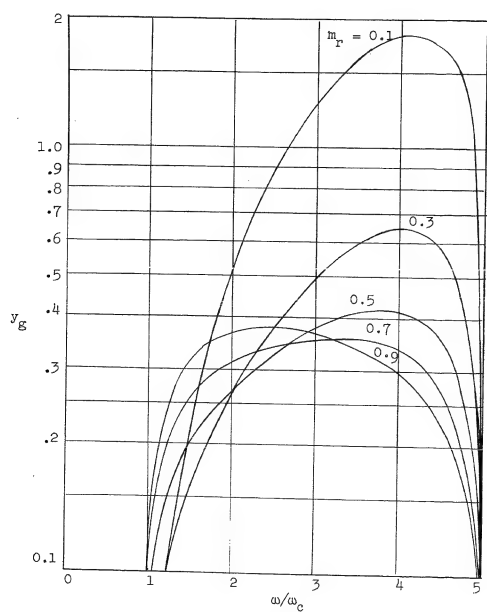


FIG. 7.13.--Group velocity y_g of the four-element band-pass circuit, $x_1=5$.

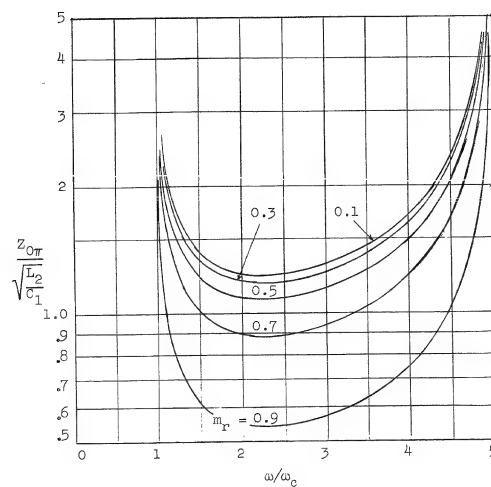


FIG. 7.14.--Circuit impedance $Z_{0\pi}$ of the four-element band-pass circuit of Pi section, $x_1=5$.

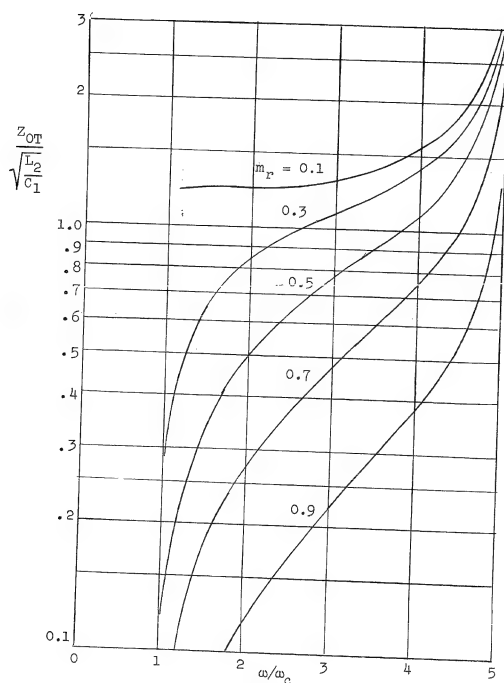


FIG. 7.15.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $x_1 = 5$.

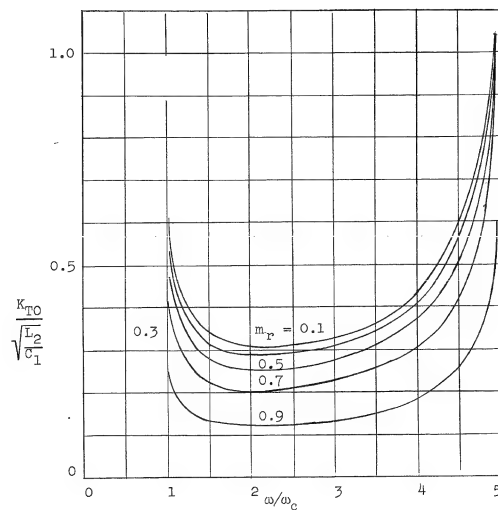


FIG. 7.16.--Transverse interaction impedance K_{T0} of the four-element band-pass circuit, $x_1 = 5$.

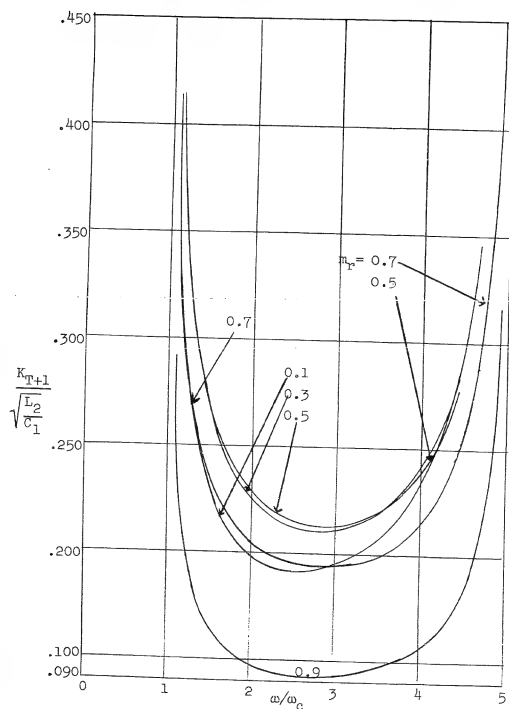


FIG. 7.17.--Transverse interaction impedance K_{T+1} of the four-element band-pass circuit, $x_1 = 5$.

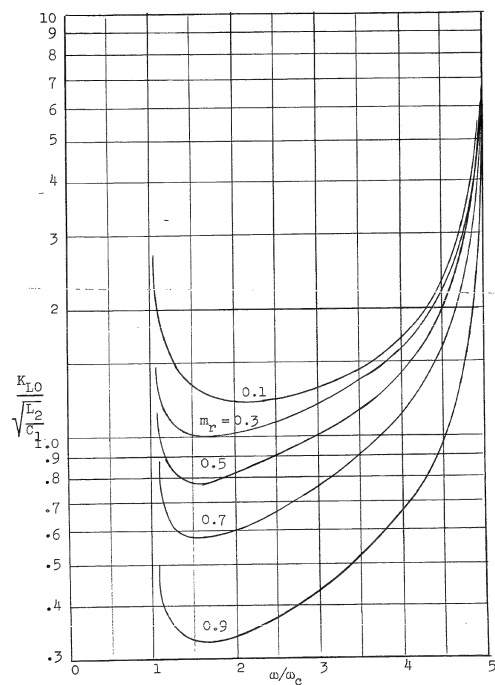


FIG. 7.18.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 5$.

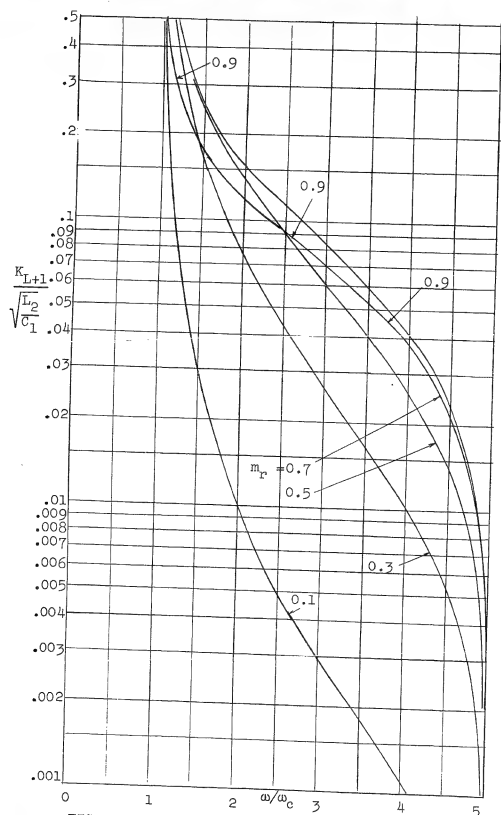


FIG. 7.19.--Longitudinal interaction impedance K_{L+1} of the four-element band-pass circuit, $x_1 = 5$.

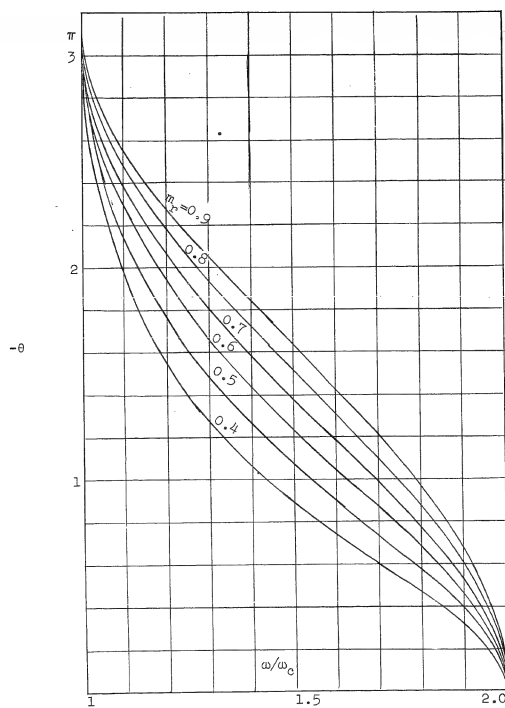
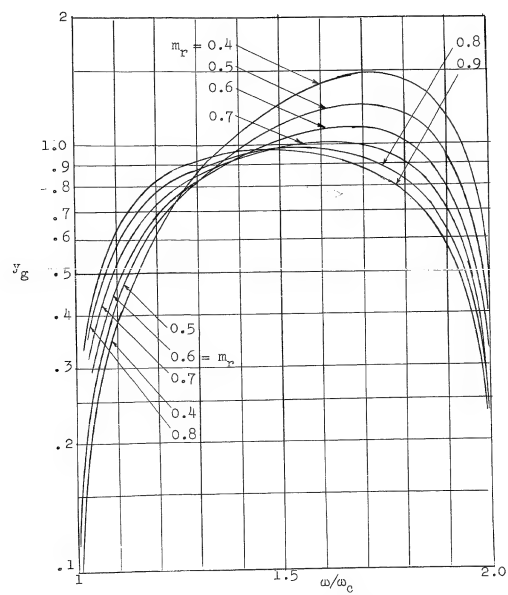
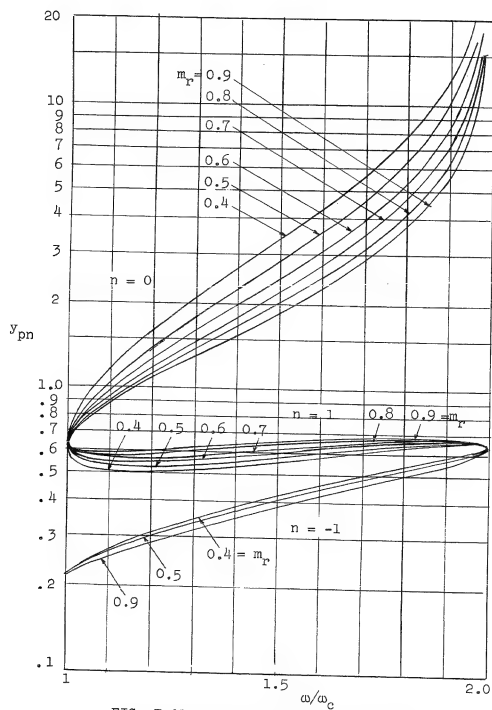


FIG. 7.20.--Phase angle per section of the four-element band-pass circuit, $x_1 = 2$.



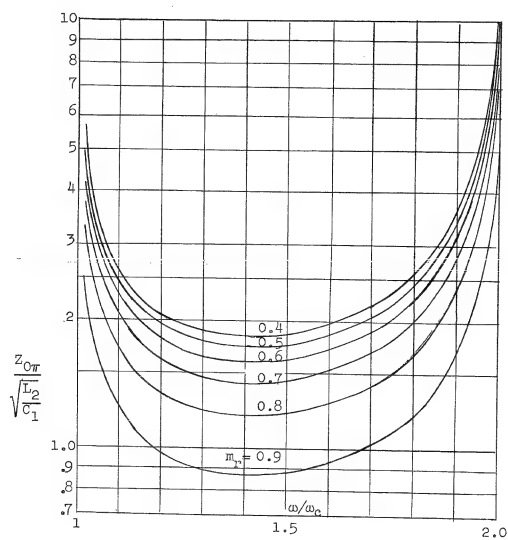


FIG. 7.23.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Pi section, $x_1 = 2$.

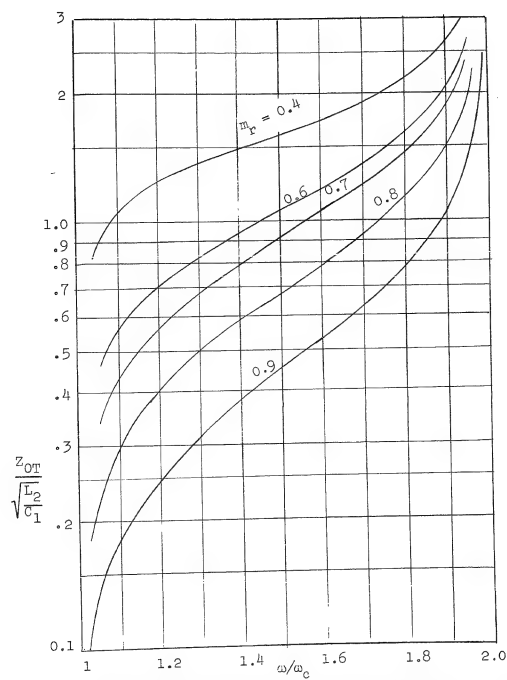


FIG. 7.24.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $x_1 = 2$.

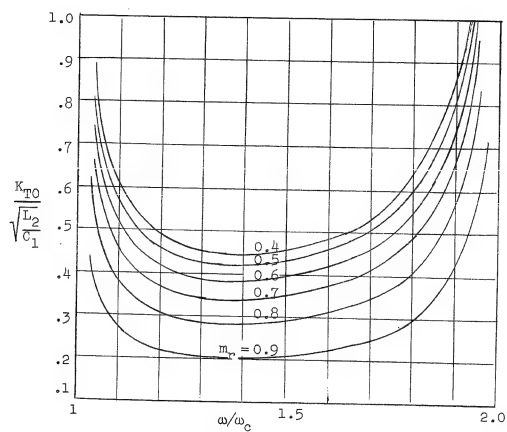


FIG. 7.25.--Transverse interaction impedance K_{T0} of the four-element band-pass circuit, $x_1 = 2$.

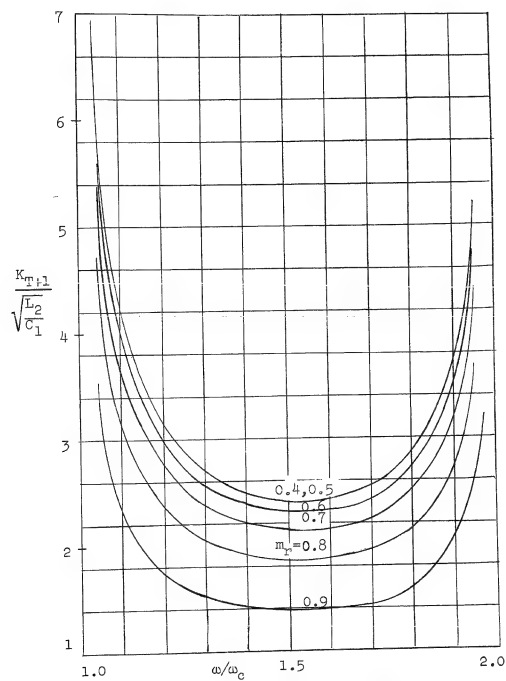


FIG. 7.26.--Transverse interaction impedance K_{T+1} of the four-element band-pass circuit, $x_1 = 2$.

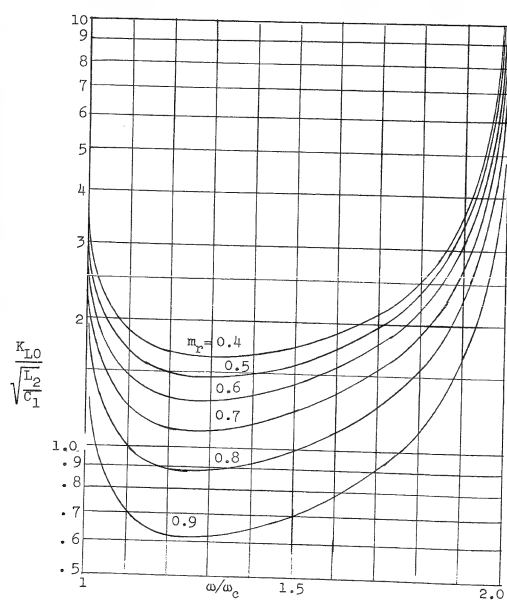


FIG. 7.27.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1=2$.

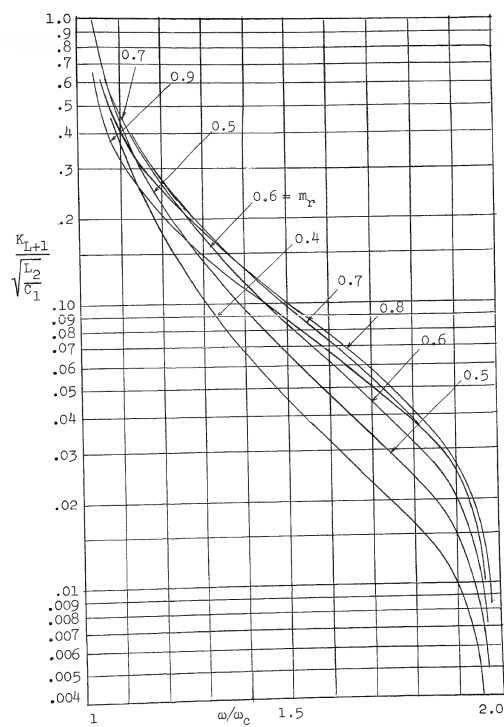


FIG. 7.28.--Longitudinal interaction impedance K_{L+1} of the four-element band-pass circuit, $x_1=2$.

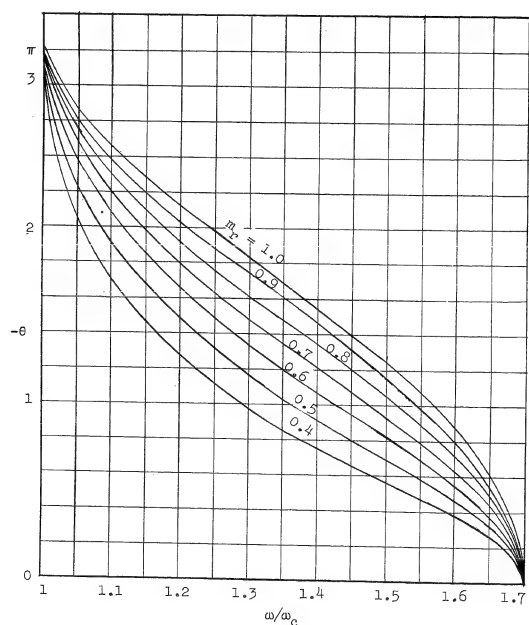


FIG. 7.29.--Phase angle per section of the four-element band-pass circuit, $x_1 = 1.70$.

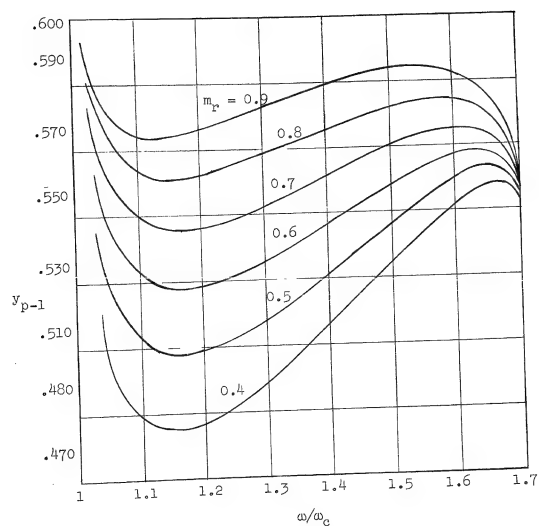


FIG. 7.30.--Phase velocity v_{p-1} of the four-element band-pass circuit, $x_1 = 1.70$.

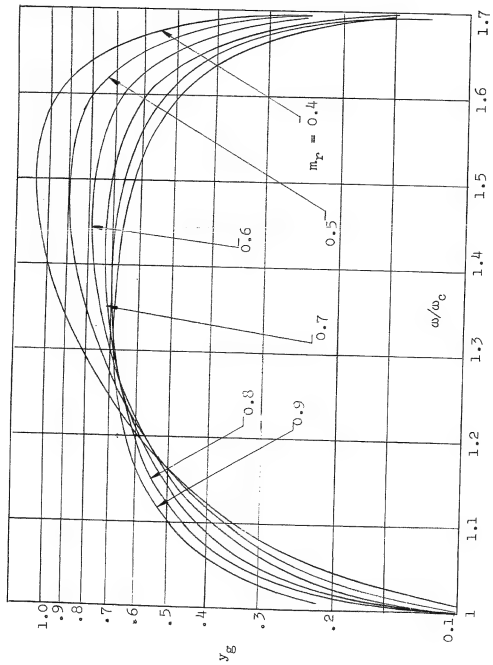


FIG. 7.31.--Group velocity y_g of the four-element band-pass circuit, $x_1=1.70$.

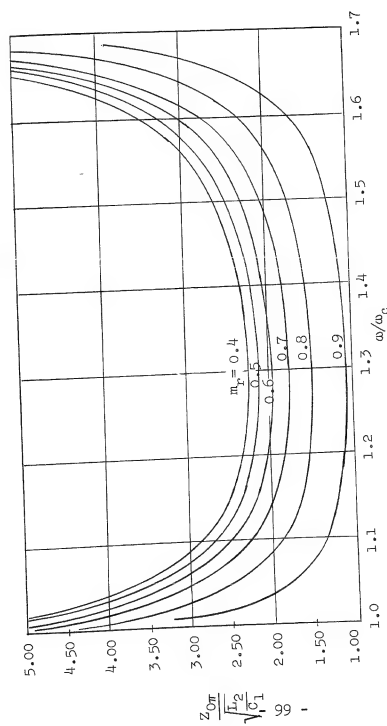


FIG. 7.32.--Circuit impedance Z_{01} of the four-element band-pass circuit of PI section, $x_1=1.70$.

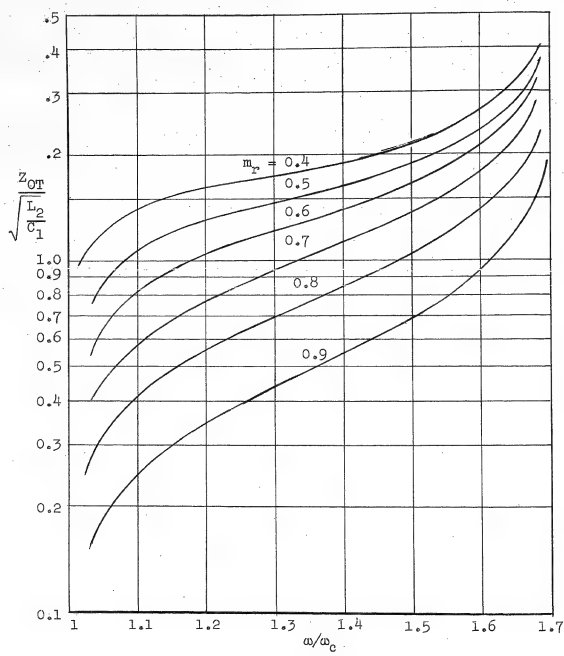


FIG. 7.33.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $X_1=1.70$.

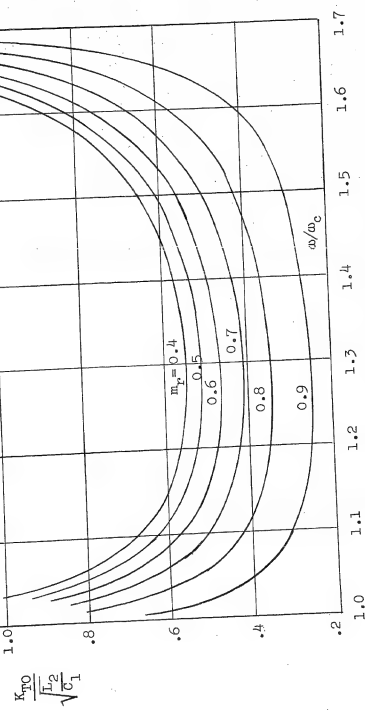


FIG. 7.34.--Transverse interaction impedance K_{OT} of the four-element band-pass circuit, $X_1=1.70$.

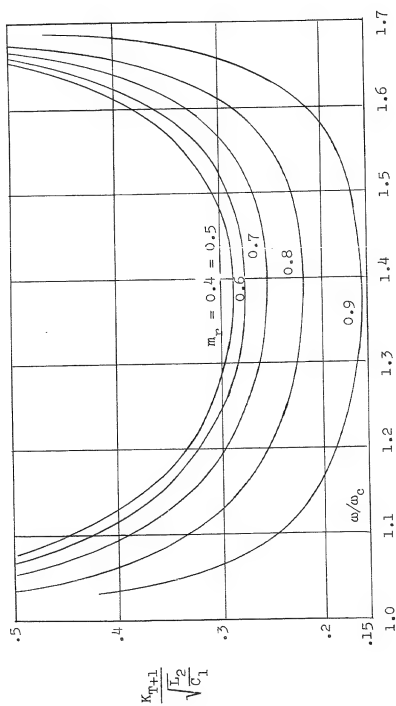


FIG. 7.35.--Transverse interaction impedance K_{T+1} of the four-element band-pass circuit, $x_1 = 1.70$.

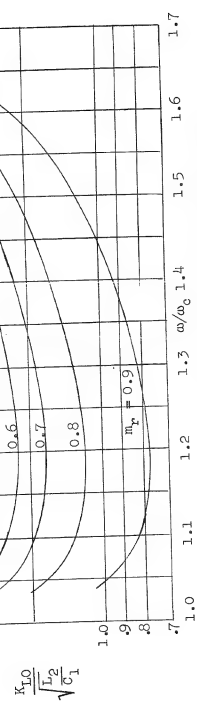
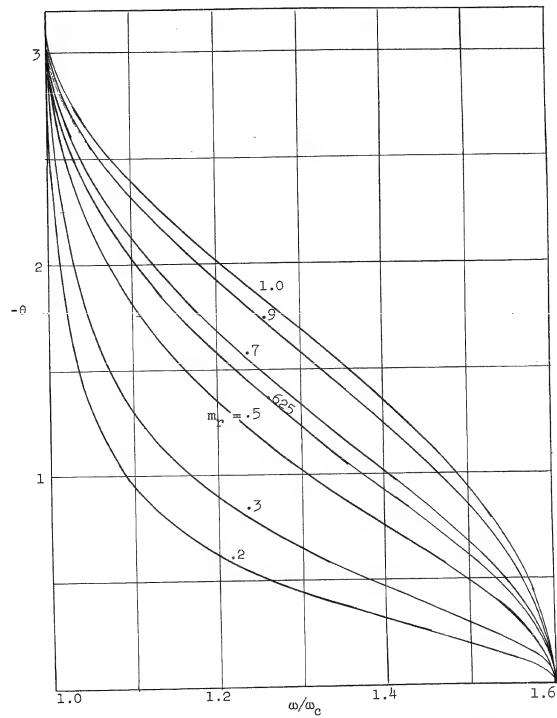
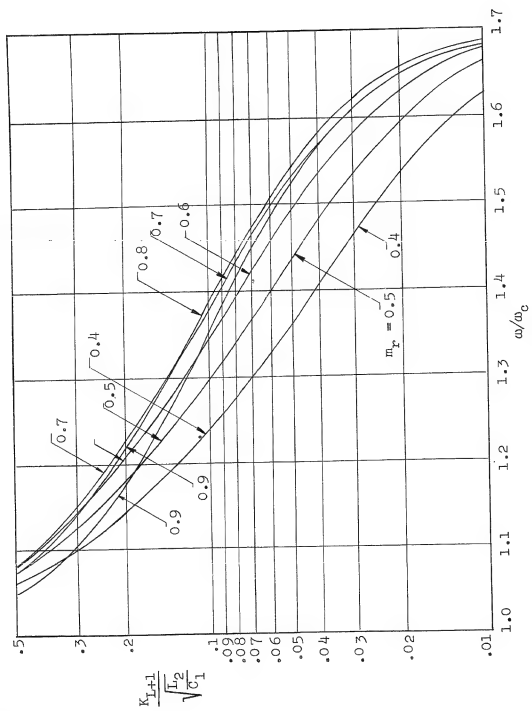


FIG. 7.36.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 1.70$.



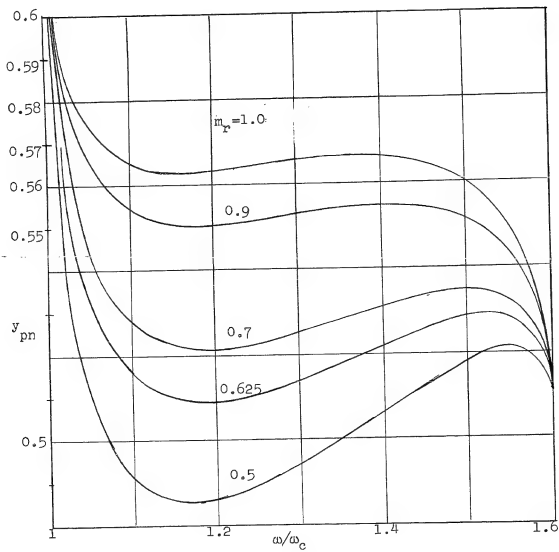


FIG. 7.39.--Phase velocity y_{pn} of the four-element band-pass circuit, $x_1 = 1.6$.

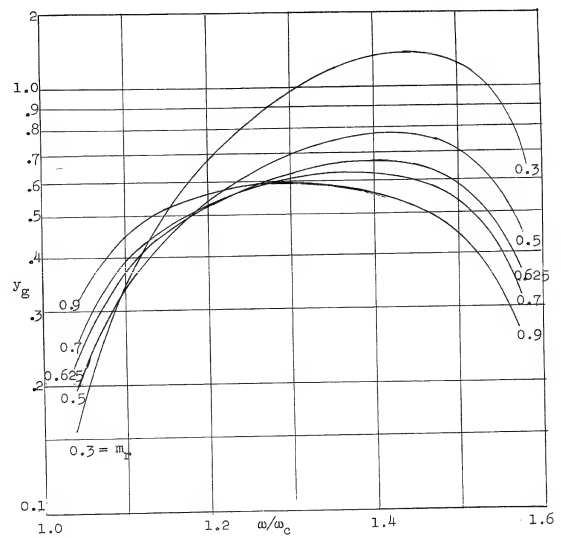


FIG. 7.40.--Group velocity y_g of the four-element band-pass circuit, $x_1 = 1.6$.

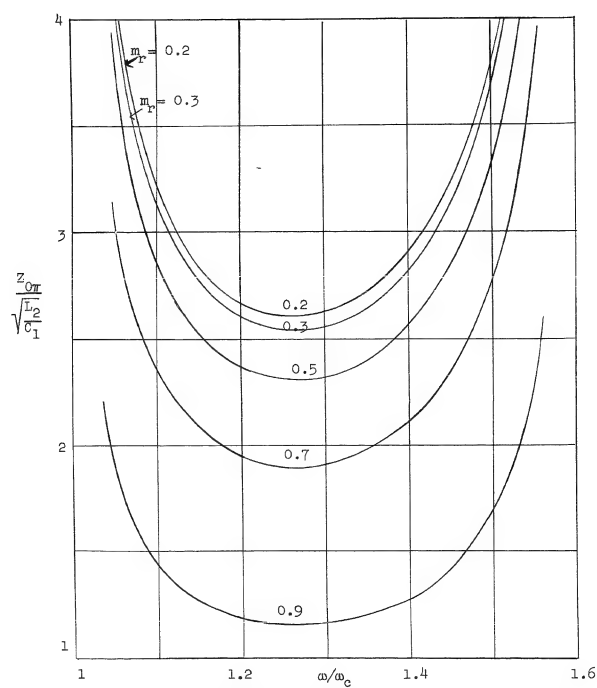


FIG. 7.41.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Pi section; $x_1 = 1.6$.

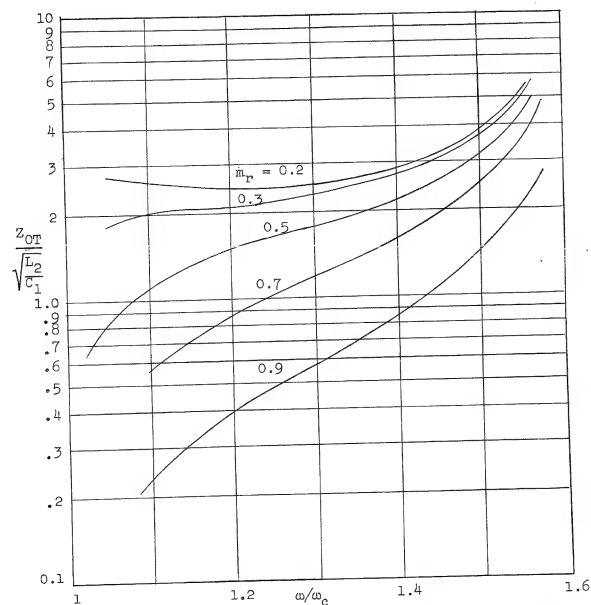


FIG. 7.42.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section; $x_1 = 1.6$.

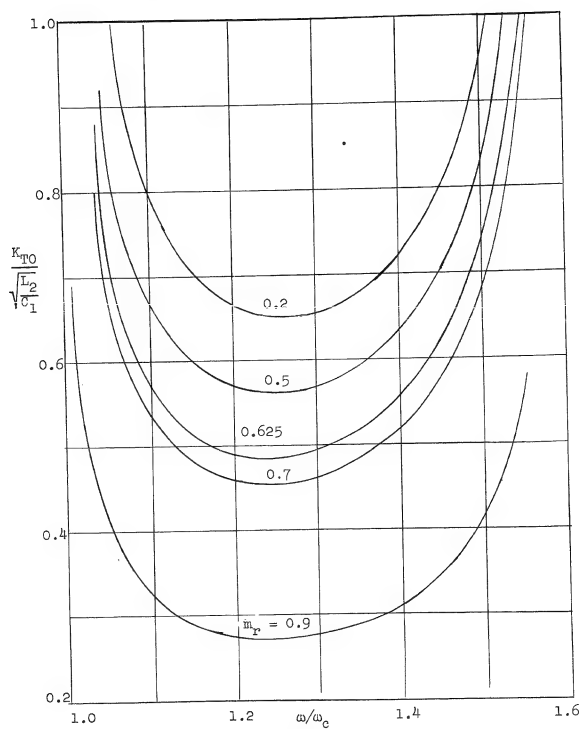


FIG. 7.43.--Transverse interaction impedance K_{T0} of the four-element band-pass circuit, $x_1=1.6$.

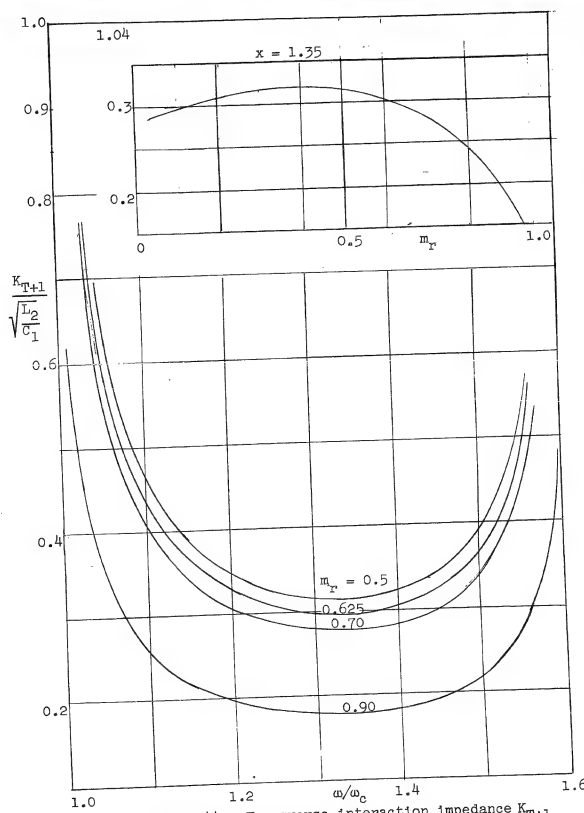


FIG. 7.44.--Transverse interaction impedance K_{T+1} of the four-element band-pass circuit, $x_1=1.6$.

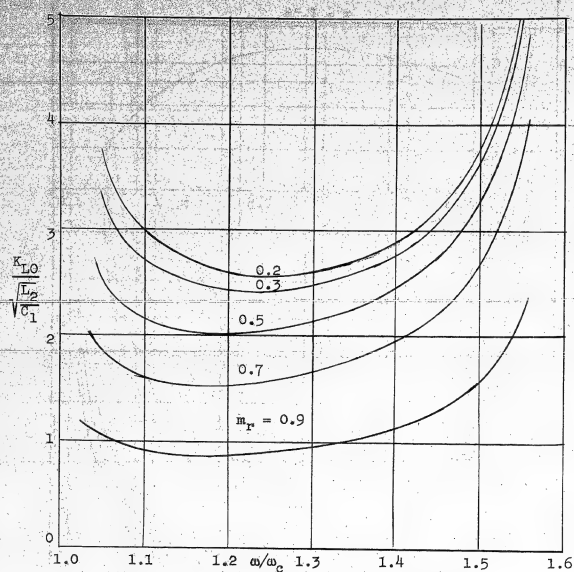


FIG. 7.45.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 1.6$.

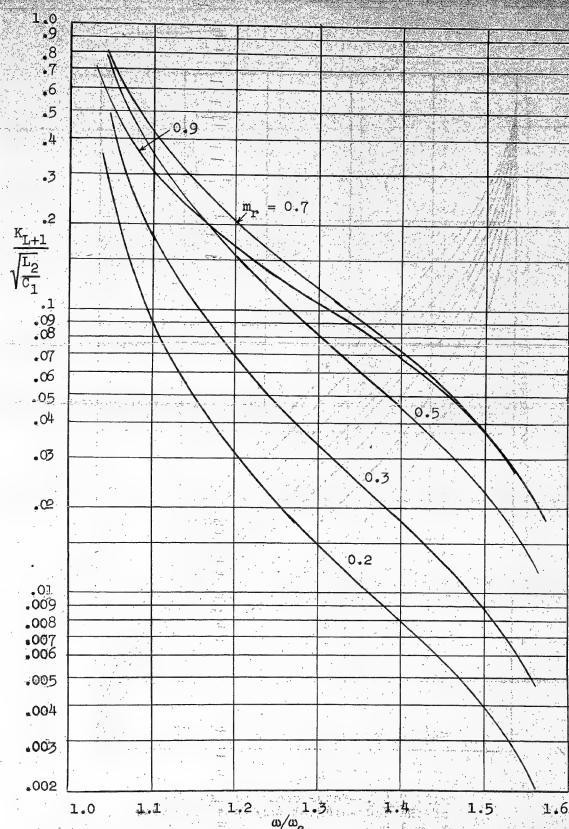


FIG. 7.46.--Longitudinal interaction impedance K_{L+1} of the four-element band-pass circuit, $x_1 = 1.6$.

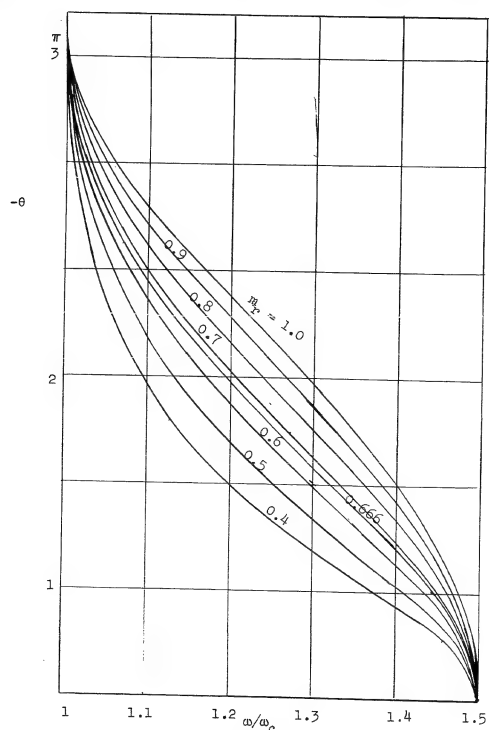


FIG. 7.47.--Phase angle per section of the four-element band-pass circuit, $x_1 = 1.5$.

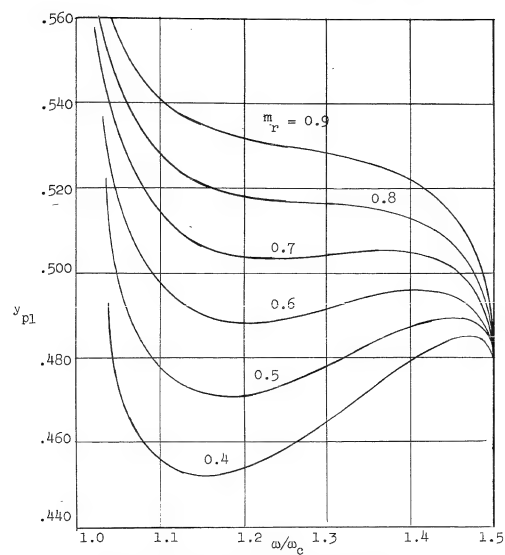


FIG. 7.48.--Phase velocity y_{p1} of the four-element band-pass circuit, $x_1 = 1.5$.

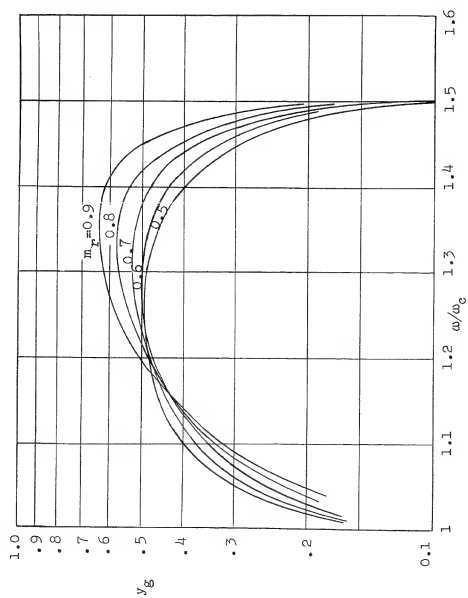


FIG. 7.49.--Group velocity y_g of the four-element band-pass circuit, $x_1 = 1.5$.

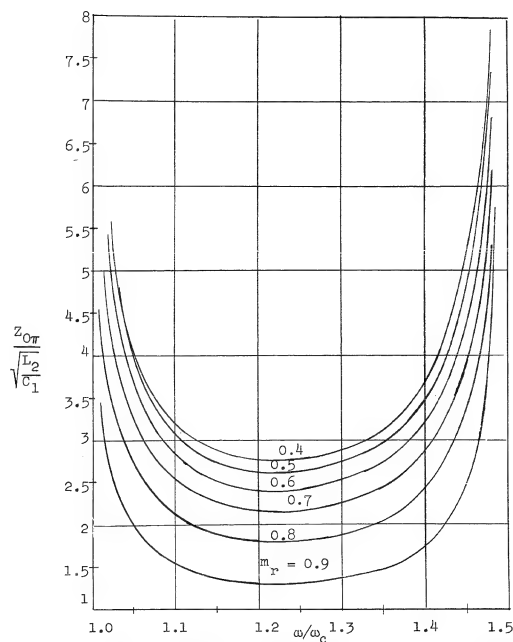


FIG. 7.50.--Circuit impedance Z_{0T} of the four-element band-pass circuit of PI section, $x_1 = 1.5$.

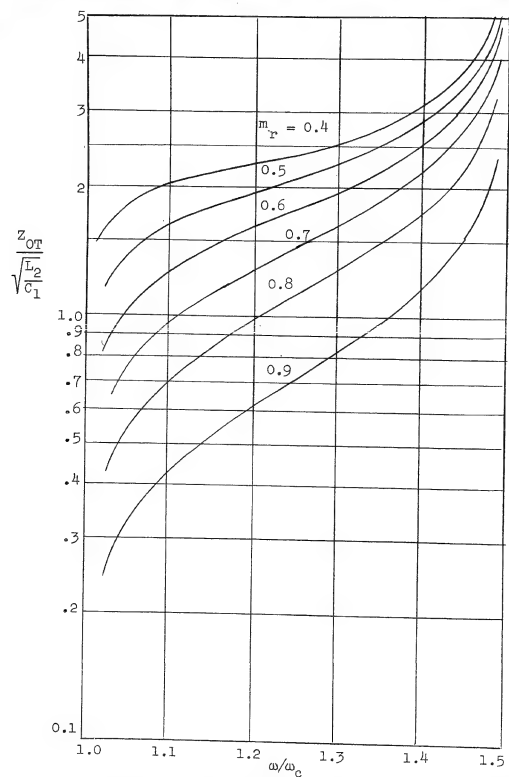


FIG. 7.51.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $x_1 = 1.5$.

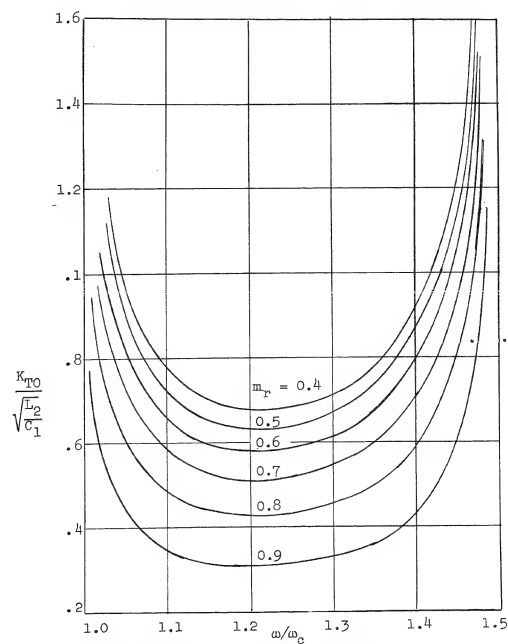


FIG. 7.52.--Transverse interaction impedance K_{TO} of the four-element band-pass circuit, $x_1 = 1.5$.

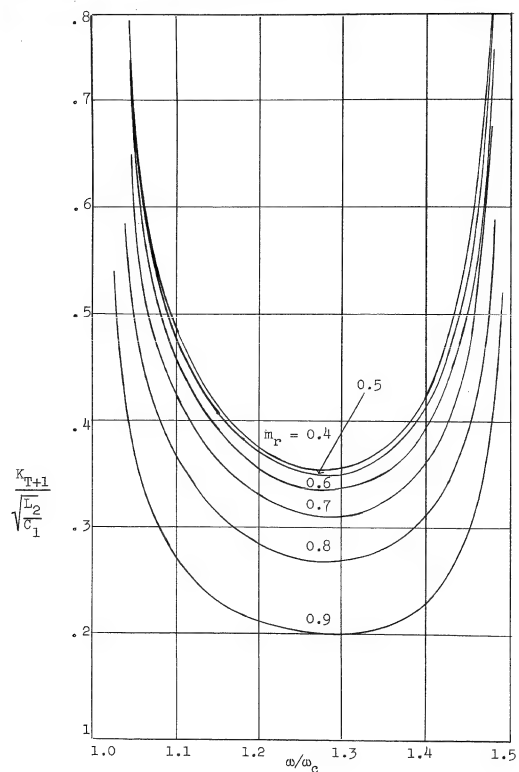


FIG. 7.53.--Transverse interaction impedance K_{T+1} of the four-element band-pass circuit, $x_1 = 1.5$.

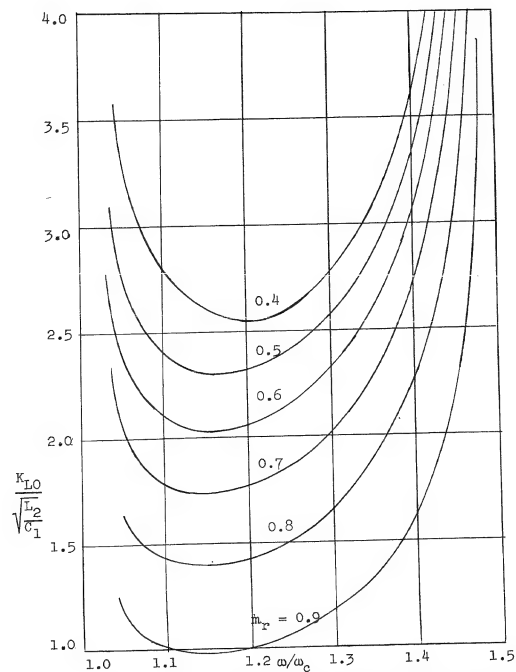


FIG. 7.54.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 1.5$.

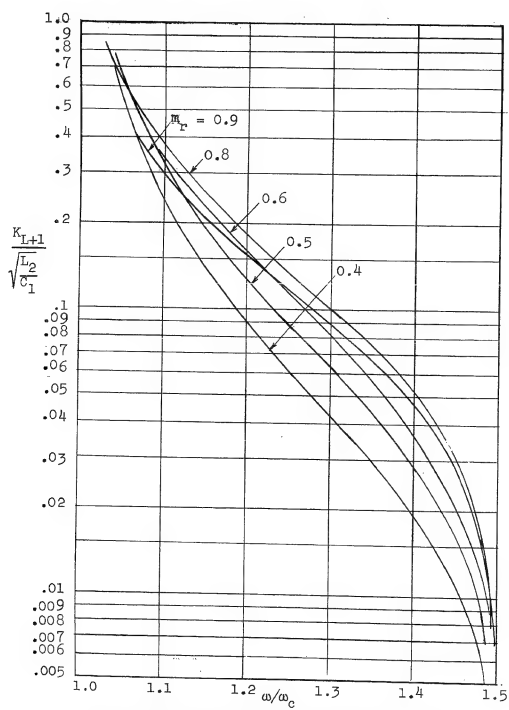


FIG. 7.55.--Longitudinal interaction impedance K_{L+1} of the four-element band-pass circuit, $x_1 = 1.5$, $L+1$

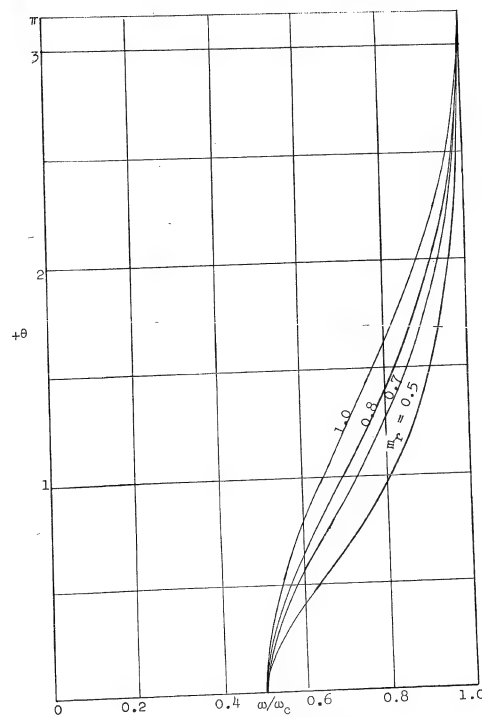


FIG. 7.56.--Phase angle per section of the four-element band-pass circuit, $x_1 = 0.5$.

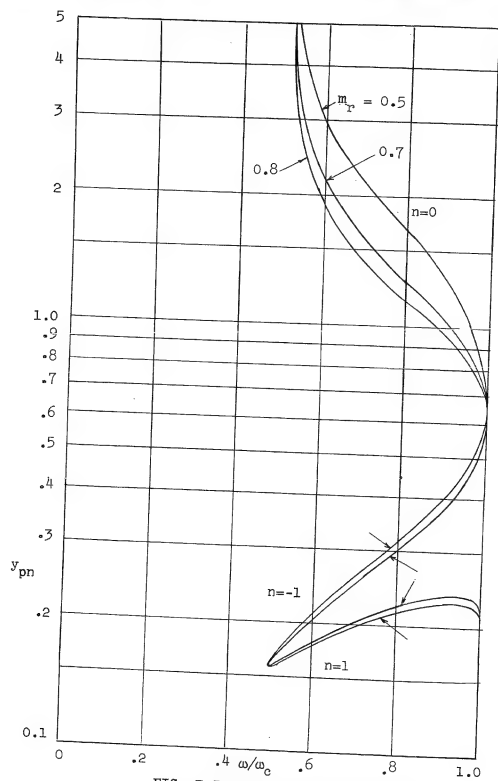


FIG. 7.57.--Phase velocity y_{pn} of the four-element band-pass circuit, $x_1 = 0.5$.

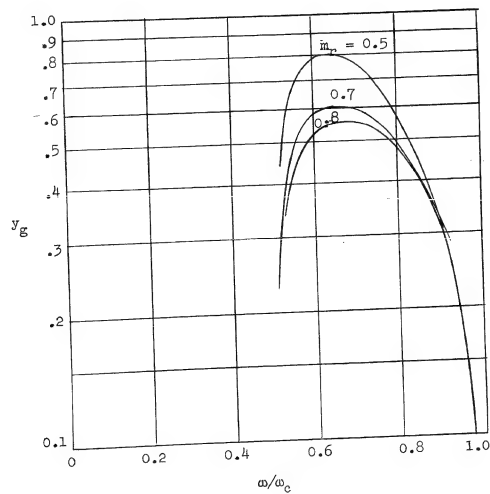


FIG. 7.58.--Group velocity y_g of the four-element band-pass circuit $x_1 = 0.5$.

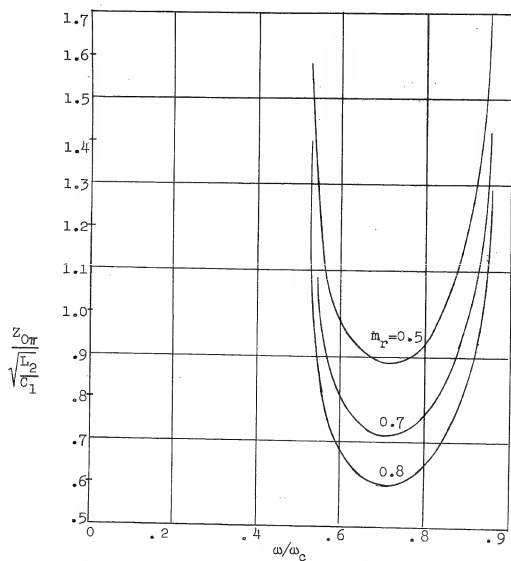


FIG. 7.59.--Circuit impedance $Z_{0\pi}$ of the four-element band-pass circuit of Pi section, $x_1 = 0.5$.

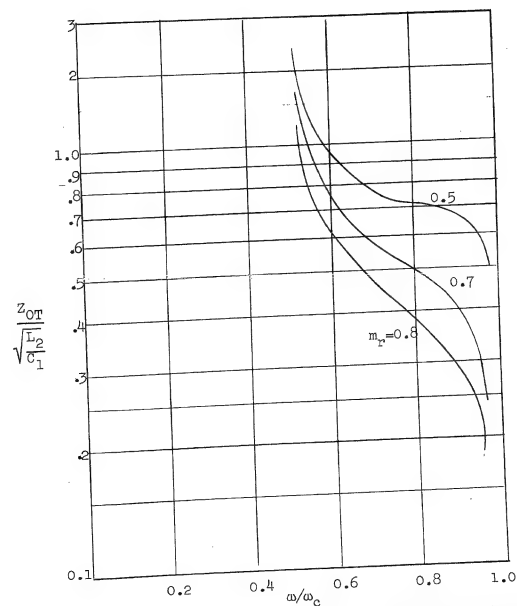
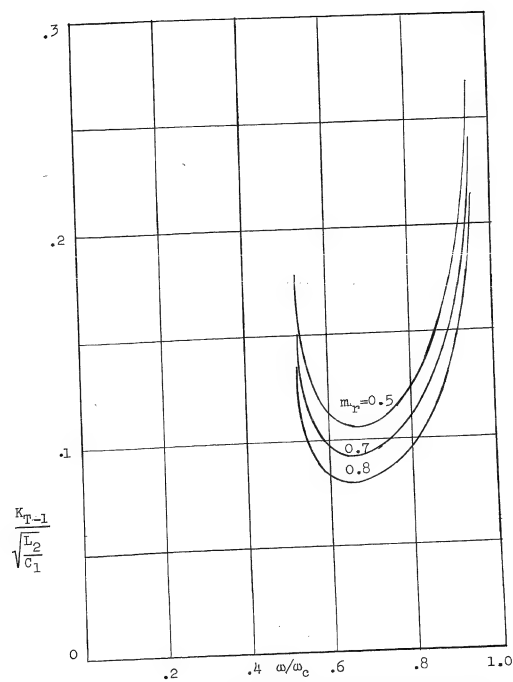
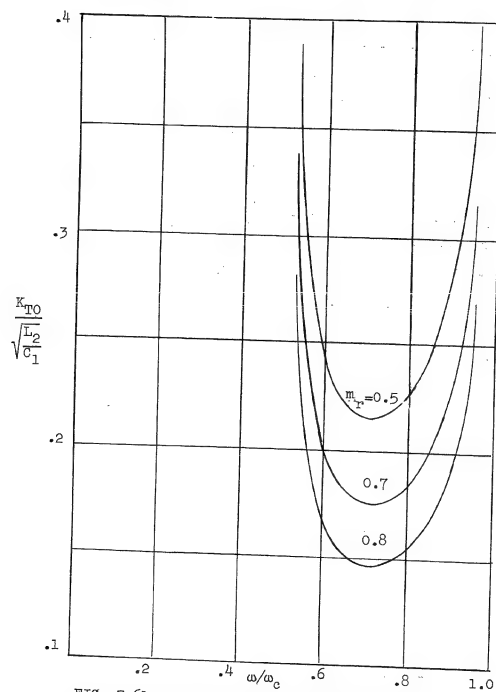


FIG. 7.60.--Circuit impedance Z_{0T} of the four-element band-pass circuit of Tee section, $x_1 = 0.5$.



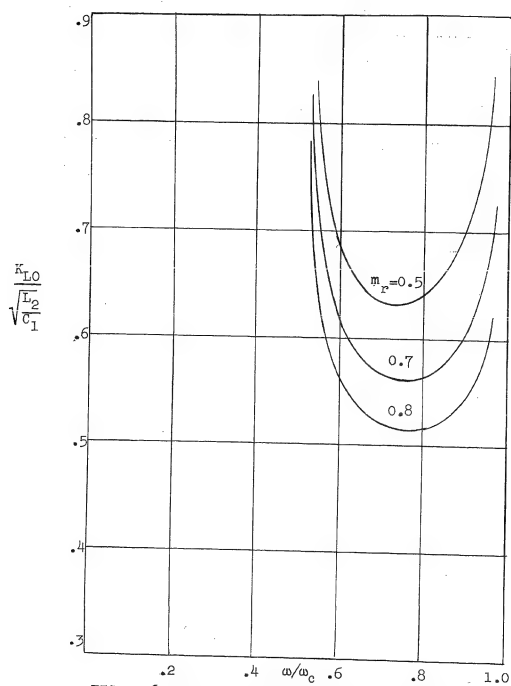


FIG. 7.63.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 0.5$.

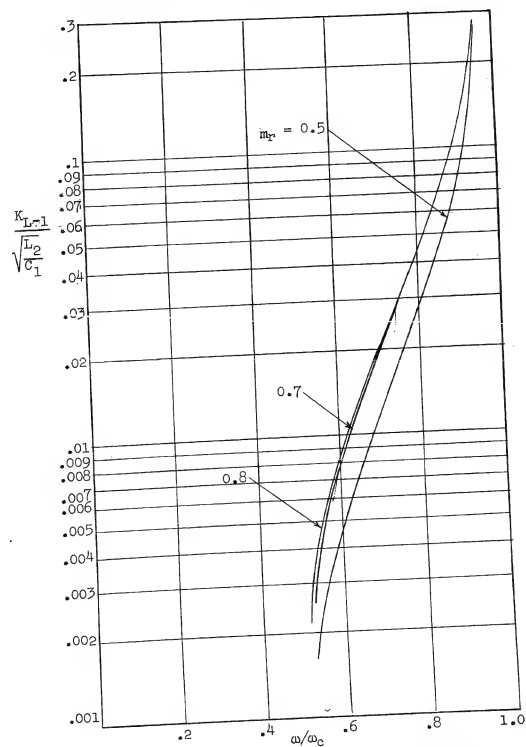


FIG. 7.64.--Longitudinal interaction impedance K_{L-1} of the four-element band-pass circuit, $x_1 = 0.5$.

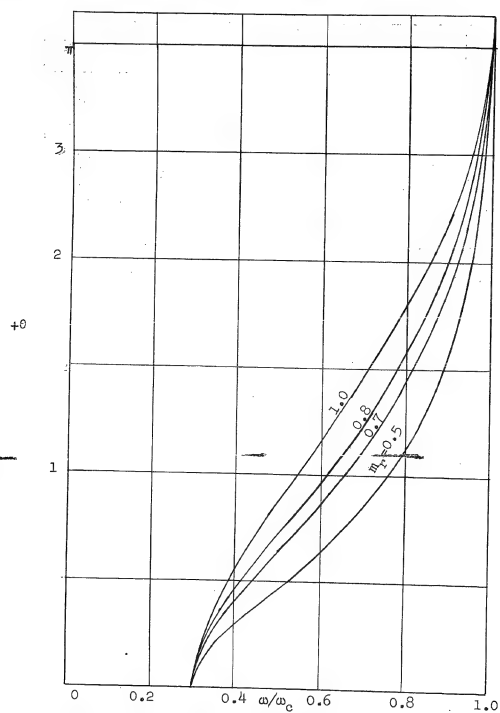


FIG. 7.65.--Phase angle per section of the four-element band-pass circuit, $x_1 = 0.3$.

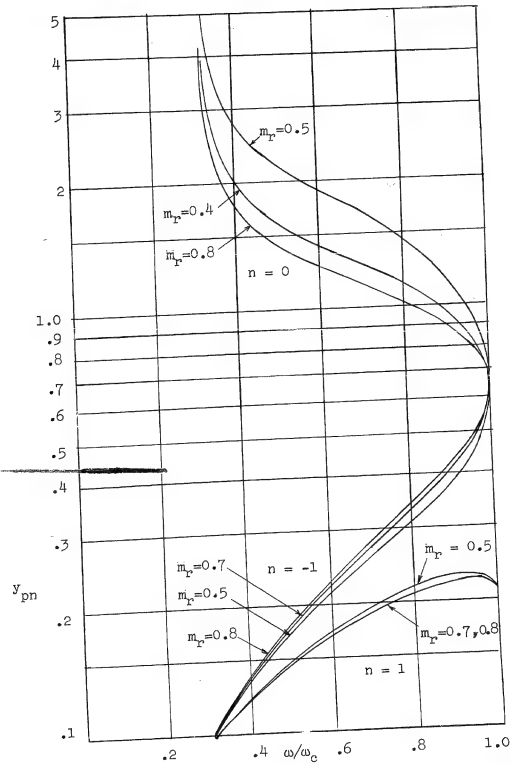


FIG. 7.66.--Phase velocity y_{pn} of the four-element band-pass circuit, $x_1 = 0.3$.

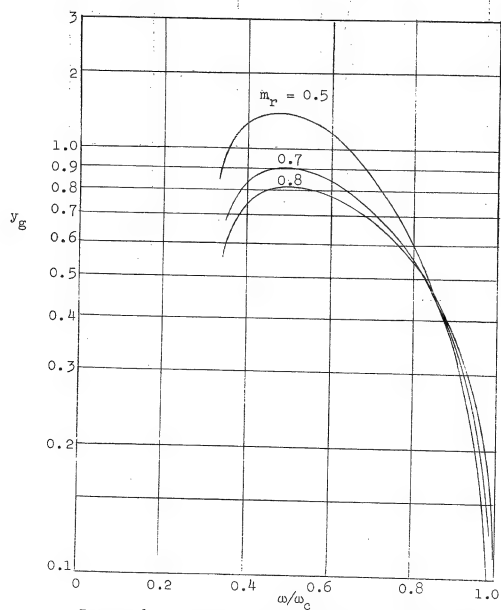


FIG. 7.67.--Group velocity v_g of the four-element band-pass circuit, $x_1 = 0.3$.

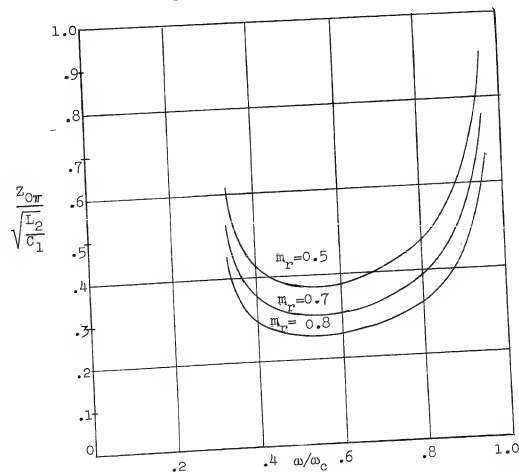


FIG. 7.68.--Circuit impedance Z_0T of the four-element band-pass circuit of Pi section, $x_1 = 0.3$.

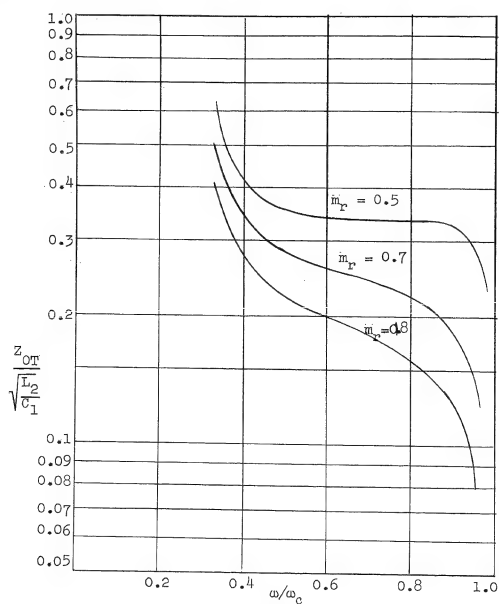


FIG. 7.69.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section, $x_1 = 0.3$.

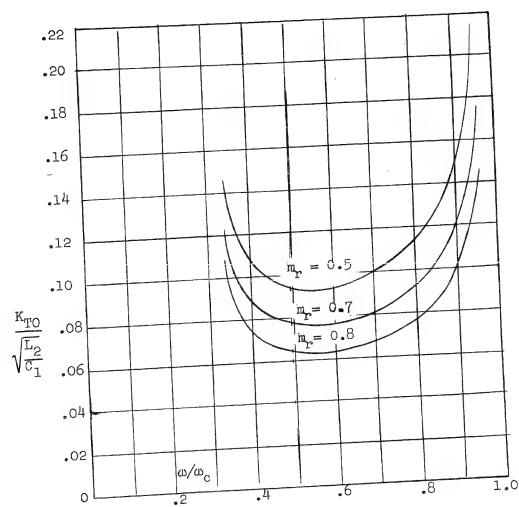


FIG. 7.70.--Transverse interaction impedance K_{TO} of the four-element band-pass circuit, $x_1 = 0.3$.

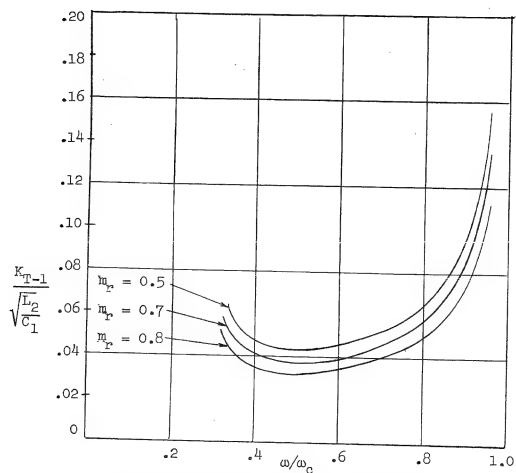


FIG. 7.71.--Transverse interaction impedance K_{T-1} of the four-element band-pass circuit, $x_1 = 0.3$.

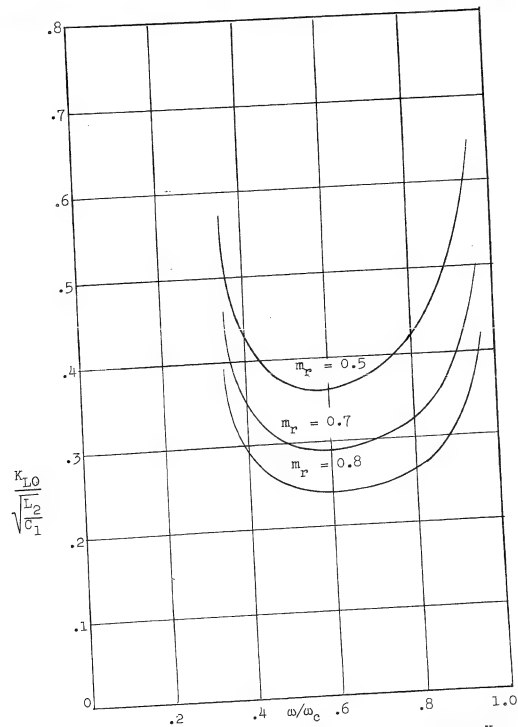


FIG. 7.72.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 0.3$.

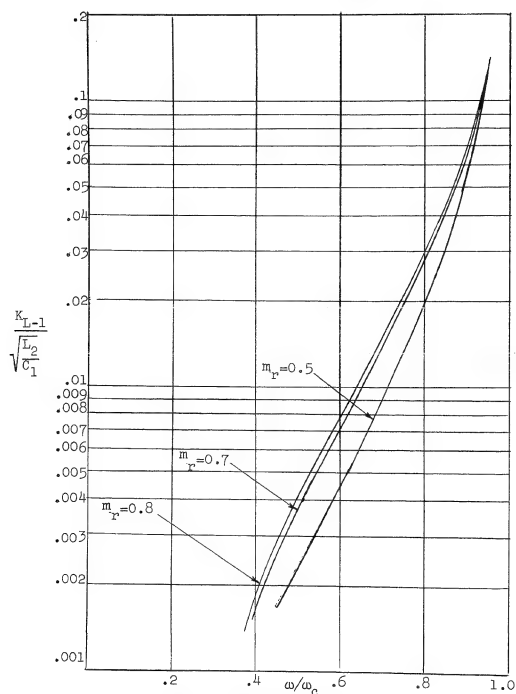


FIG. 7.73.--Longitudinal interaction impedance K_{L-1} of the four-element band-pass circuit $x_1 = 0.3$.

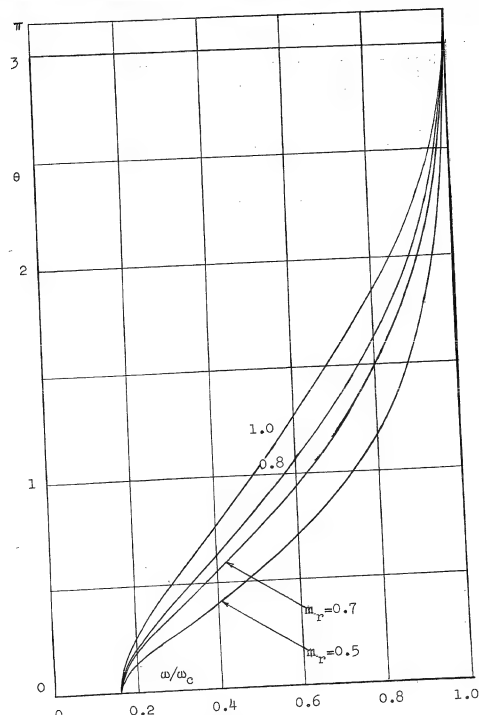


FIG. 7.74.--Phase angle per section of the four-element band-pass circuit, $x_1 = 0.16$.

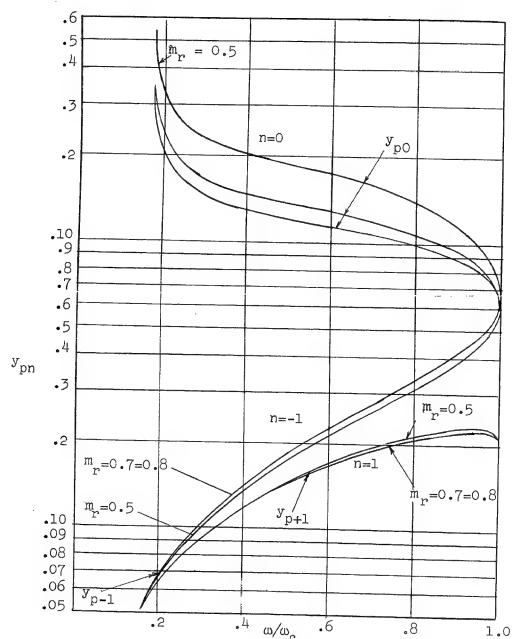


FIG. 7.75.--Phase velocity v_{pn} of the four-element band-pass circuit, $x_1 = 0.16$.

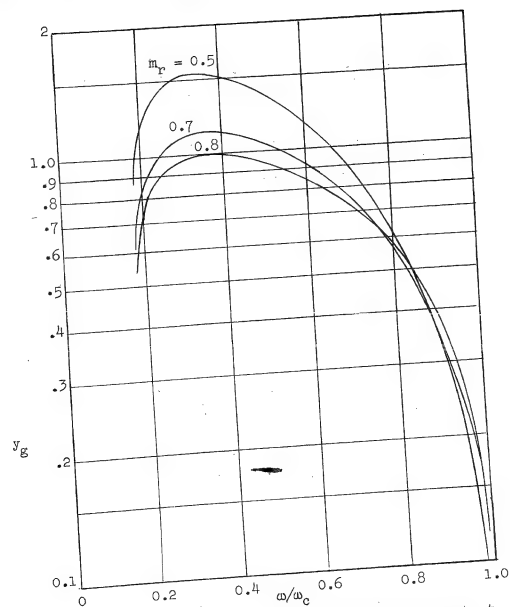


FIG. 7.76.--Group velocity v_g of the four-element band-pass circuit, $x_1 = 0.16$.

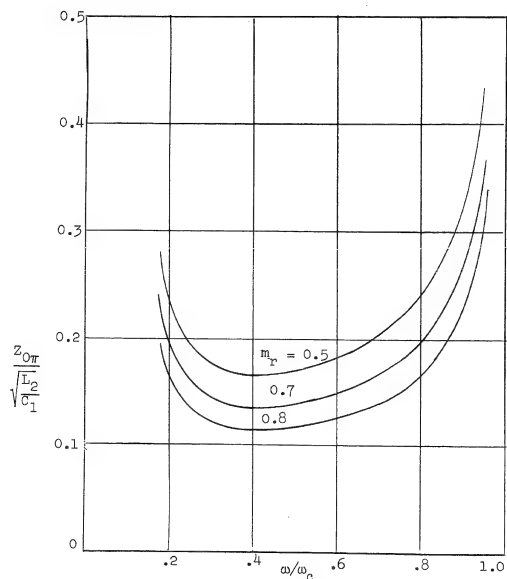


FIG. 7.77.--Circuit impedance Z_{0T} of the four-element band-pass circuit of Pi section, $x_1 = 0.16$.

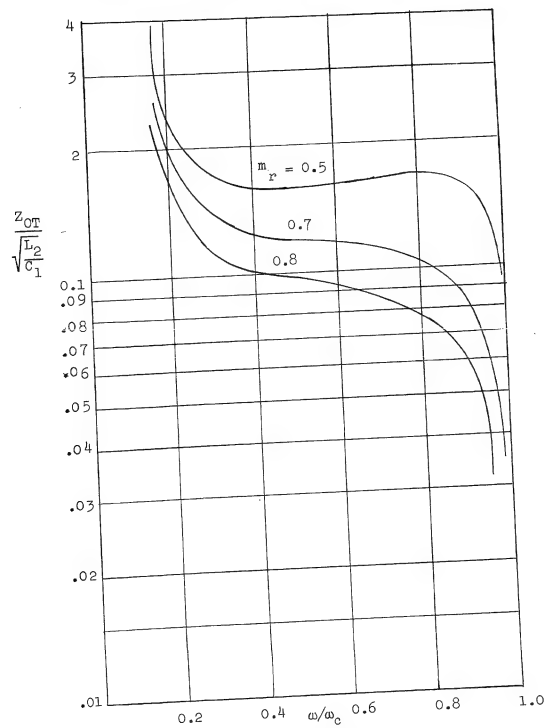
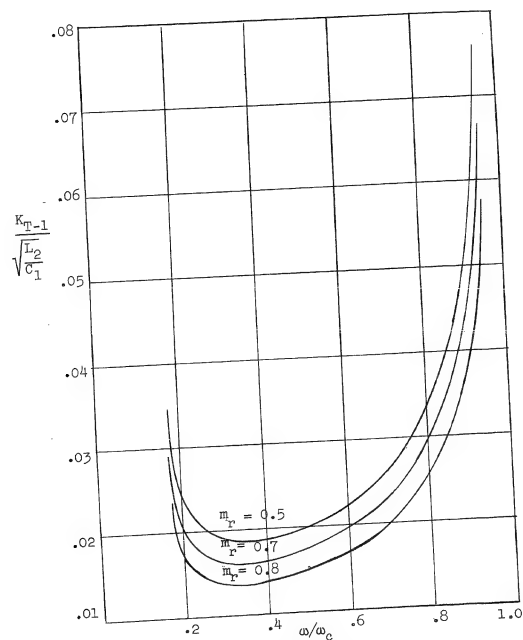
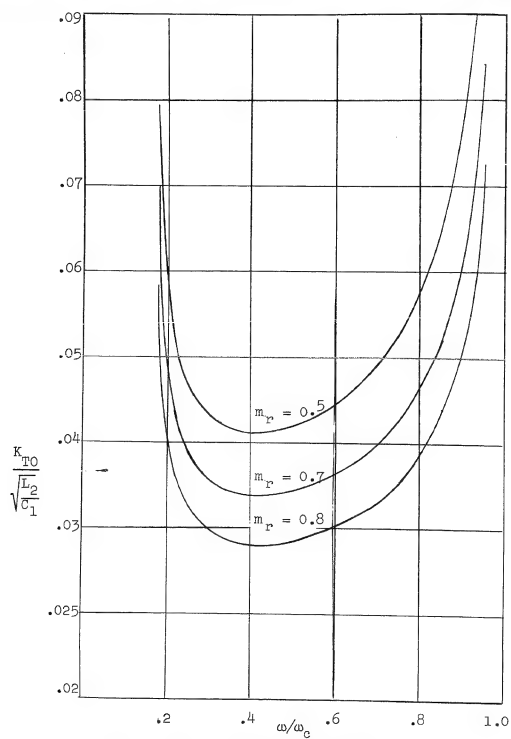


FIG. 7.78.--Circuit impedance Z_{0T} of the four-element band-pass circuit of Tee section $x_1 = 0.16$.



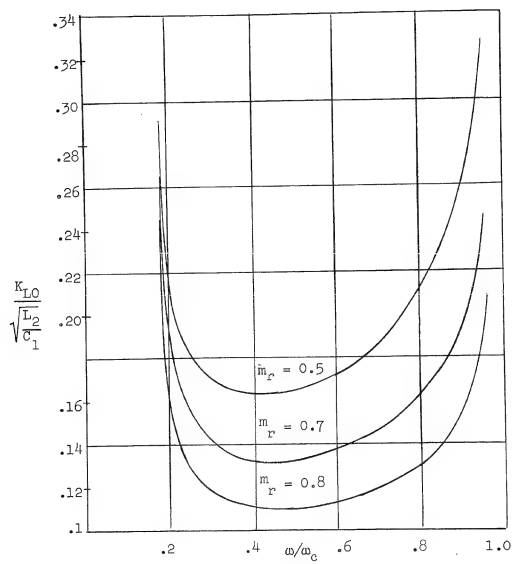


FIG. 7.81.--Longitudinal interaction impedance K_{L0} of the four-element band-pass circuit, $x_1 = 0.16$.

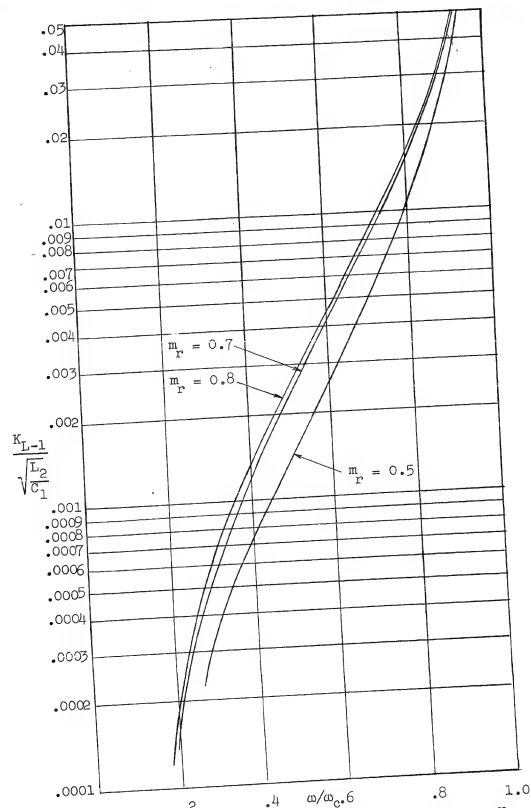


FIG. 7.82.--Longitudinal interaction impedance K_{L-1} of the four-element band-pass circuit, $x_1 = 0.16$.

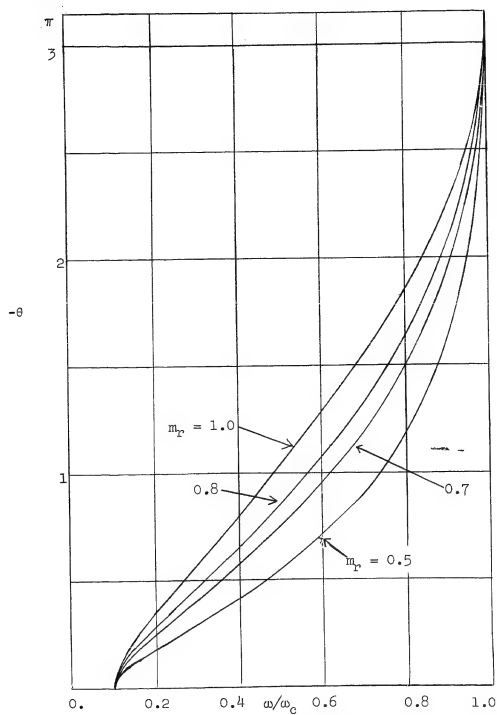


FIG. 7.83.--Phase angle per section of the four-element band-pass circuit, $x_1 = 0.100$.

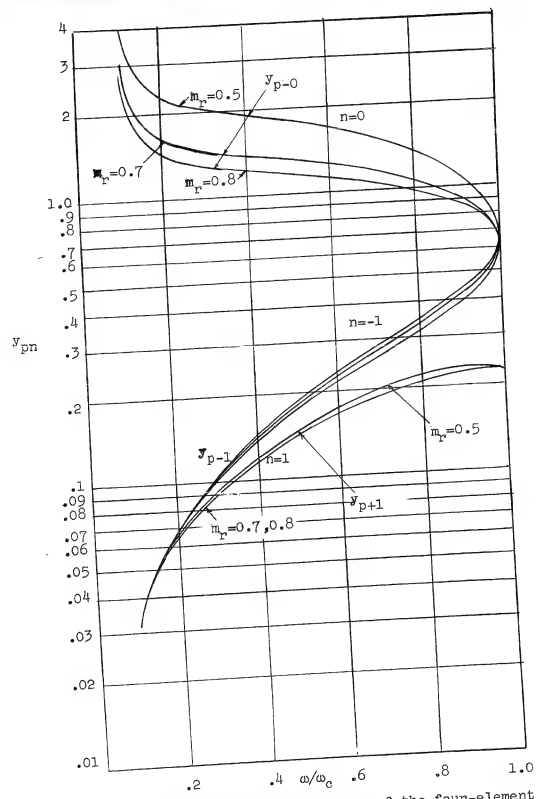


FIG. 7.84.--Phase velocity v_{pn} of the four-element band-pass circuit, $x_1 = 0.100$.

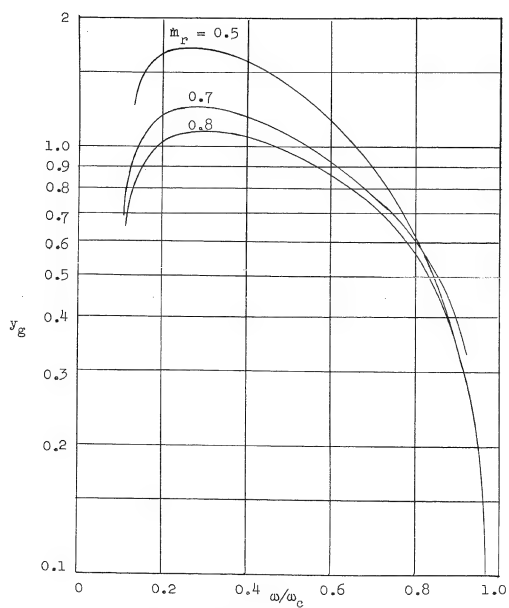


FIG. 7.85.--Group velocity y_g of the four-element band-pass circuit, $x_1 = 0.100$.

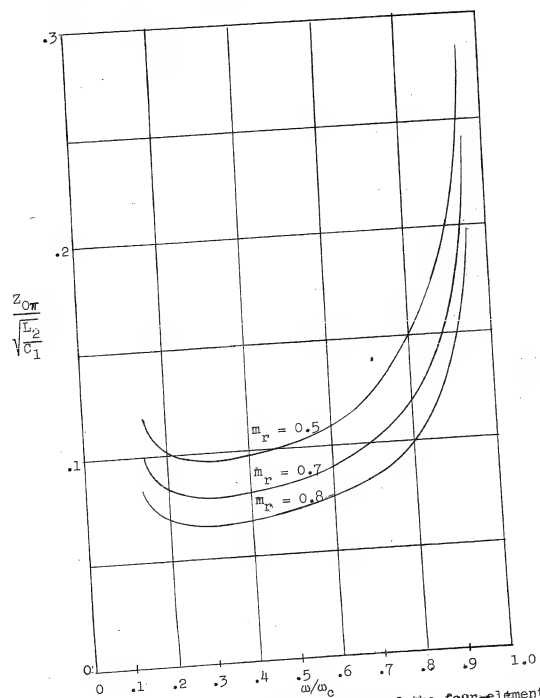


FIG. 7.86.--Circuit impedance Z_{0T} of the four-element band-pass circuit of P1 section, $x_1 = 0.100$.

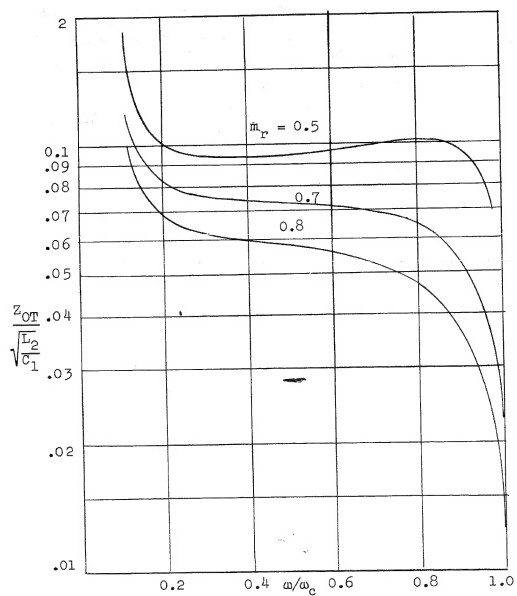


FIG. 7.87.--Circuit impedance Z_{OT} of the four-element band-pass circuit of Tee section $x_1 = 0.100$.

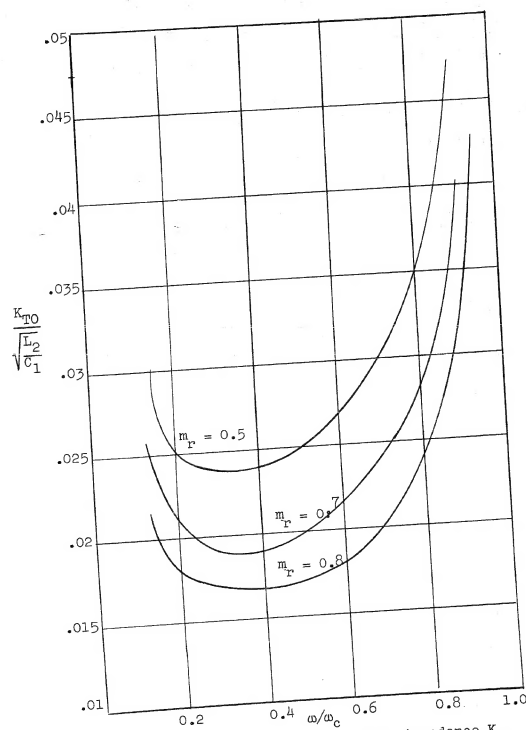
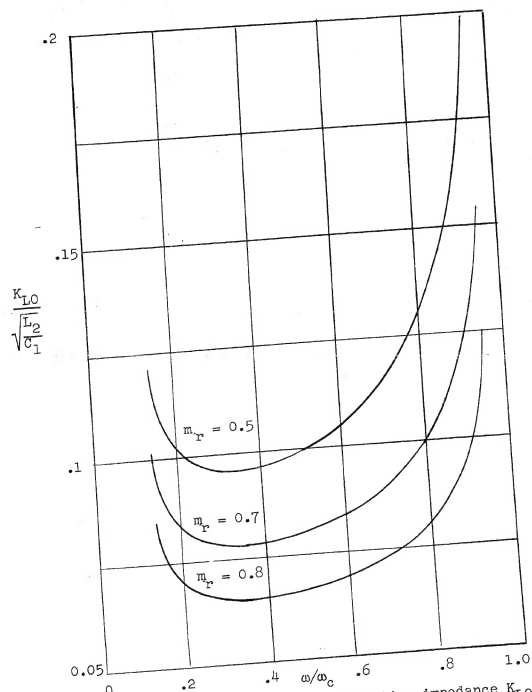
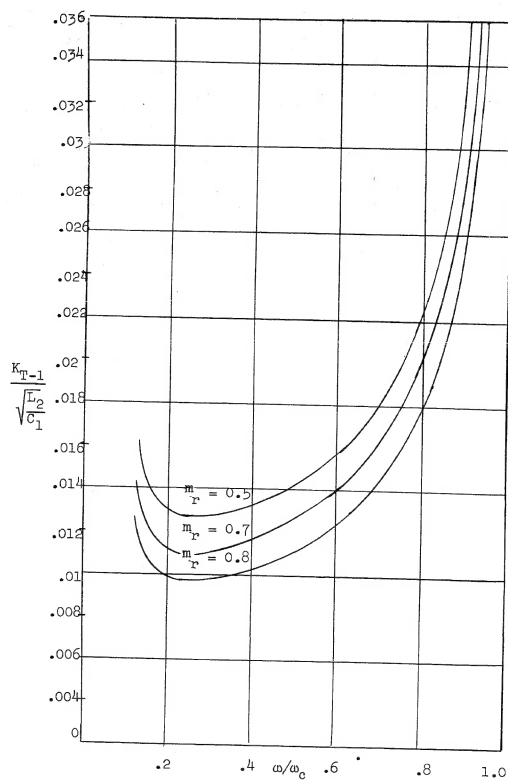


FIG. 7.88.--Transverse interaction impedance K_{T0} of the four-element band-pass circuit, $x_1 = 0.100$.



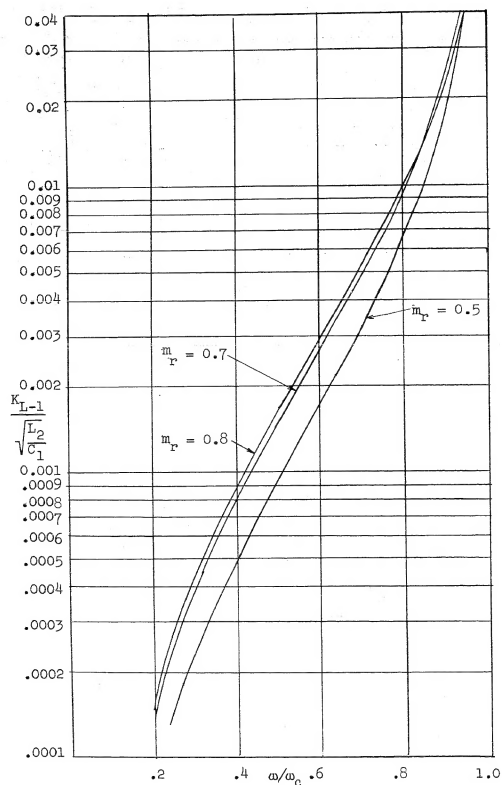


FIG. 7.91.--Longitudinal interaction impedance K_{L-1} of the four-element band-pass circuit, $x_1=0.001$.

REFERENCES

1. C. T. Sah, "Analysis for external-circuit traveling-wave tubes," T.R. No. 1, Nonr 225 (24), Stanford Electronics Laboratories, Stanford University, California.

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